CS333 - Work Sheet 1

1. If you measure the state $|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + i\sqrt{\frac{2}{3}}|1\rangle$ in the standard basis, what happens, and with what probability? What about if you measure using the basis $\{|\rightarrow\rangle, |\leftarrow\rangle\}$?

   Solution With the first measurement, you get outcome $|0\rangle$ with probability 1/3 and outcome $|0\rangle$ with probability 2/3. With the second measurement, you get outcome $|\rightarrow\rangle$ with probability $\frac{(\sqrt{2}-1)^2}{6}$, and outcome $|\leftarrow\rangle$ with probability $\frac{(\sqrt{2}+1)^2}{6}$.

2. If you have two qubits states $|\psi\rangle$ and $|\phi\rangle$ such that $\langle\psi|\phi\rangle = 0$, explain what measurement you should use to perfectly distinguish between these two states?

   Solution Note that $\{|\psi\rangle, |\phi\rangle\}$ is an orthonormal basis, since $\langle\psi|\psi\rangle = \langle\phi|\phi\rangle = 1$ because $|\phi\rangle$ and $|\psi\rangle$ represent qubit states, and $\langle\phi|\psi\rangle = 0$. So $\{|\psi\rangle, |\phi\rangle\}$ represents a valid quantum meausurement. If we use this measurement to measure either $|\phi\rangle$ or $|\psi\rangle$, we will get each outcome with probability 1 or 0, so we perfectly distinguish between these states.

3. Explain what will happen in the following situations using bra and ket notation, and the language of collapse:

   (a) Alice prepares a right diagonal photon, and Bob measures it using a vertically polarized filter.

   (b) Alice prepares a left diagonal photon, and Bob measures it using a right diagonally polarized filter.

   Solution

   (a) Alice prepares the state $|+\rangle$, and Bob measures in the $\{|0\rangle, |1\rangle\}$ basis. Therefore, he gets outcome $|0\rangle$ with probability $|\langle 0|+\rangle|^2 = 1/2$. In this case, the qubit collapses to the state $|0\rangle$ and so passes through the filter. He gets outcome $|1\rangle$ with probability $|\langle 1|+\rangle|^2 = 1/2$. In this case, the state collapses to the state $|1\rangle$ and gets blocked by the filter.

   (b) Alice prepares the state $|-\rangle$, and Bob measures in the $\{|+\rangle, |-\rangle\}$ basis. Therefore, he gets outcome $|-\rangle$ with probability $|\langle -|-\rangle|^2 = 1$. In this case, the qubit doesn't collapse (it stays in the same state), which gets blocked by the filter, so nothing emerges.