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Entanglement: can have 2-qubit states that are not tensor product

$$|\psi\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle = \begin{pmatrix} a_{00} \\ a_{01} \\ a_{10} \\ a_{11} \end{pmatrix}$$

is quantum state iff $|a_{00}|^2 + |a_{01}|^2 + |a_{10}|^2 + |a_{11}|^2 = \langle\psi|\psi\rangle = 1$

(amplitudes square to 1)

<u>def</u>: A state $|\psi\rangle$ is <u>product</u> if $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$

<u>def</u>: A state $|\psi\rangle$ is <u>entangled</u> if $\nexists$ $|\psi_1\rangle, |\psi_2\rangle$ such that $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$

Q: • Let $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$

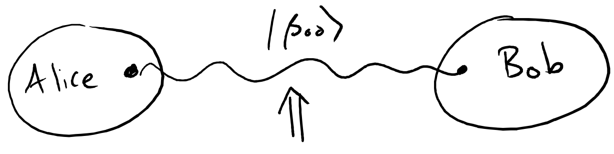
Is $|\beta_{00}\rangle$ entangled?

Yes: $\begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix}$

$ad = 0$ $\quad a \neq 0$ b/c $ac \neq 0$

So $d = 0$. But contradiction $\rightarrow bd \neq 0$

Scenario:



Entangled State of 2 qubits.

Neither qubit has a definite state on its own, but together, they have a well defined state

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Alice and Bob can't communicate, but they can make a quantum measurement on their subsystem.

$$\underbrace{\begin{array}{ll}\text{Alice Measures} & M_A = \{|\phi_0\rangle, |\phi_1\rangle\} \\ \text{Bob Measures} & M_B = \{|\chi_0\rangle, |\chi_1\rangle\}\end{array}}$$

Effective measurement on $|\psi\rangle_{AB}$ (their combined state):

$$M_{AB} = M_A \otimes M_B = \{|\phi_0\rangle|\chi_0\rangle, |\phi_1\rangle|\chi_0\rangle, |\phi_0\rangle|\chi_1\rangle, |\phi_1\rangle|\chi_1\rangle\}$$

- Get outcome $\underset{\text{Alice's outcome}}{|\phi_i\rangle_A}\,\underset{\text{Bob's outcome}}{|\chi_j\rangle_B}$ with probability $\left|\langle\psi|_{AB}|\phi_i\rangle_A|\chi_j\rangle_B\right|^2$
- State $|\psi\rangle_{AB} \rightarrow |\phi_i\rangle_A|\chi_j\rangle_B$ (collapse)

$$\boxed{\text{If } |\psi\rangle_{AB} \text{ was entangled} \rightarrow \text{collapses to unentangled}}$$

Measurement destroys/uses up entanglement

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Let

$$M(\theta) = \{ |\phi_0(\theta)\rangle, |\phi_1(\theta)\rangle \}$$

$$|\phi_0(\theta)\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$$
$$|\phi_1(\theta)\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

If $x=0$, Alice measures $M(0)$ on her qubit
If $x=1$, Alice measures $M(\pi/4)$ on her qubit

If $y=0$, Bob measures $M(\pi/8)$ on his qubit

If $y=1$, Bob measures $M(-\pi/8)$ on his qubit
If Alice or Bob gets outcome $|\phi_0\rangle$, returns 0, otherwise returns 1 to referee

Case 1: $x=0, y=0$

$$M = \{ |0\rangle_A|\phi_0(\pi/8)\rangle_B, |0\rangle_A|\phi_1(\tfrac{\pi}{8})\rangle_B, |1\rangle_A|\phi_0(\pi/8)\rangle_B, |1\rangle_A|\phi_0(\pi/8)\rangle_B \}$$

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00: $\left| \langle\phi_0(0)|_A \langle\phi_0(\pi/8)| \left(\frac{1}{\sqrt{2}}|00\rangle_{AB} + \frac{1}{\sqrt{2}}|11\rangle\right)\right|^2$

$= \frac{1}{2}\left| \underbrace{\langle\phi_0(0)|0\rangle}_{1} \underbrace{\langle\phi_0(\pi/8)|0\rangle}_{\cos\pi/8} + \underbrace{\langle\phi_0(0)|1\rangle}_{0} \langle\phi_0(\pi/8)|1\rangle \right|$

$= \frac{1}{2}\left|\cos\left(\frac{\pi}{8}\right)\right|^2$

01: $\frac{1}{2}\left|\langle\phi_0(0)|0\rangle\langle\phi_1(\pi/8)|0\rangle\right|^2$

$= \frac{1}{2}\left|\sin(\pi/8)\right|^2$

10: $\frac{1}{2}\left| \underbrace{\langle\phi_1(0)|0\rangle}_{0} \langle\phi_0(\pi/8)|0\rangle + \underbrace{\langle\phi_1(0)|1\rangle}_{1} \underbrace{\langle\phi_0(\pi/8)|1\rangle}_{\sin\pi/2} \right|^2$

$= \frac{1}{2}\left|\sin\pi/8\right|^2$

11: $\frac{1}{2}\left|\cos\pi/8\right|^2$

$x \wedge y = 0$, so want $a \oplus b = 0$, want $a=b=0$, $a=b=1$.

These could outcomes occur with probability $\cos(\pi/8)^2 = .853$