CS333

1. Suppose you would like to transform a qubit in the state $|0\rangle$ into a qubit in the state $|1\rangle$. There are many unitaries that can accomplish this. Describe the complete set of unitaries that accomplish this transformation both in terms of the Bloch sphere and in terms of a matrix representation.

2. Prove that

$$C_p = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{1}$$

   is an entangling gate. By entangling gate, I mean it is possible to have this unitary act on an unentangled state, and have the result be an entangled state.

3. (This problem has a lot more calculation than I would have you do on the exam, but it is good practice.) Here is another quantum game. This time with three players, Alice, Bob and Charlie. The referee sends a bit x to Alice, y to Bob, and z to Charlie. Alice returns a bit a, Bob a bit b, and Charlie a bit c. The referee either gives all players 0, or the referee gives two players 1's and one player a 0. The players can not communicate. They win if $a \oplus b \oplus c = x \vee y \vee z$. Here is a table of the winning conditions:

xyz	$a \oplus b \oplus c$
000	0
011	1
101	1
110	1

(2)

   (a) Describe a deterministic (non-probabilistic) classical strategy that has as large a winning probability as possible (averaged over the choice of x, y, and z) and determine what the chance of winning is. (Probabilistic strategies can't do better than deterministic strategies for these games, so restricting to deterministic strategies doesn't hurt us.)

   (b) Suppose Alice, Bob, and Charlie share the 3-qubit quantum state $|\psi\rangle = \frac{1}{2}|000\rangle - \frac{1}{2}|011\rangle - \frac{1}{2}|101\rangle - \frac{1}{2}|110\rangle$. Prove that this state is entangled. That is, prove that there is no tensor product of three single qubit states that equals $|\psi\rangle$.

   (c) Consider the strategy that if a player receives a 0, they measure using the basis $\{|0\rangle, |1\rangle\}$, and return 0 with outcome $|0\rangle$ and 1 with outcome $|1\rangle$, if a player receives a 1, they measure using the basis $\{|+\rangle, |-\rangle\}$, and return 0 with outcome $|+\rangle$ and 1 with outcome $|-\rangle$

      i. If $x = y = z = 0$, what is their success probability?

ii. If $x = 0$, $y = z = 1$, what is their success probability? (By symmetry, this case is the same as the remaning cases.)