CS302 - Problem Set 12

Here are some problem definitions:

- k-INDSET: Given an undirected, unweighted graph $G = (V, E)$, is there a set $V' \subseteq V$ such that $|V'| \geq k$, and for all $v, u \in V'$, there is no edge $\{u, v\} \in E$? (If yes, output the set.)

- k-CLIQUE: Given an undirected, unweighted graph $G = (V, E)$, is there a set $V' \subseteq V$ such that $|V'| \geq k$, and for all $v, u \in V'$, there is an edge $\{u, v\} \in E$? (If yes, output the set.)

- DOUBLE-SAT: Given a CNF formula with at most l clauses, where l is a polynomial, involving the variables $x_1, x_2, \ldots, x_n$ and their negations, are there at least two different satisfying solutions? For example, $(x_1 \vee \neg x_1 \vee \neg x_2) \wedge (x_2 \vee x_3) \wedge (\neg x_3)$ has two valid assignments, $x_1 = 1, x_2 = 1, x_3 = 0$ and $x_1 = 0, x_2 = 1, x_3 = 0$. (If yes, output two solutions.)

1. [**6 points**] Explain why any problem in NP (using our definition from class) can be solved in exponential (i.e. $O(2^{n^k})$) time where n is the size of the input, and k is a positive constant.

    These are challenging. Start early and get the gears churning.

2. (**) Prove DOUBLE-SAT is NP-Complete.

    (a) [**11 points**] Prove DOUBLE-SAT is in NP.

    (b) Prove DOUBLE-SAT is NP-Hard

        i. [**6 points**] Describe a reduction from 3-SAT to DOUBLE-SAT, and show the reduction takes polynomial time.

        ii. [**11 points**] Prove that there is a solution to the 3-SAT problem if and only if there is a solution to the DOUBLE-SAT problem.

3. Show that k-CLIQUE reduces to k-INDSET.

    (a) [**6 points**] Describe the conversion from a k-CLIQUE input to a to k-INDSET input and explain why it takes polynomial time.

    (b) [**6 points**] Explain why there is a solution to the original k-CLIQUE problem if and only if there is a solution to your k-INDSET problem.

4. Show that 3-SAT reduces to k-INDSET.

    (a) [**6 points**] Describe the conversion from any 3-SAT input to a k-INDSET input, and explain why it takes polynomial time.

    (b) [**6 points**] Explain why there is a solution to the 3-SAT problem if and only if there is a solution to your k-INDSET problem.