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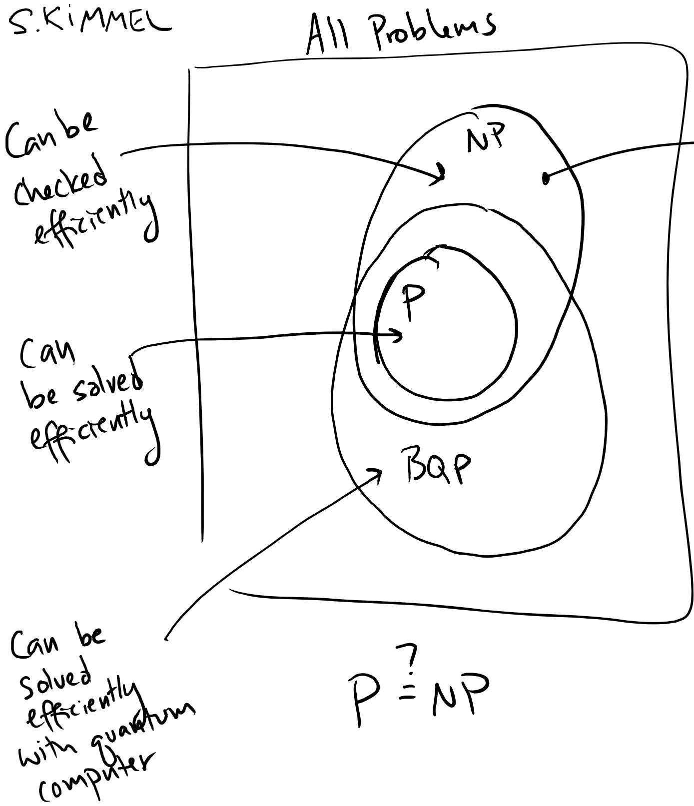


$P \stackrel{?}{=} NP$

TSP:

Travelling Salesperson

Is there a path with distance $\leq C$?

- In NP
- We think not in P, but don't know

Famous, b/c

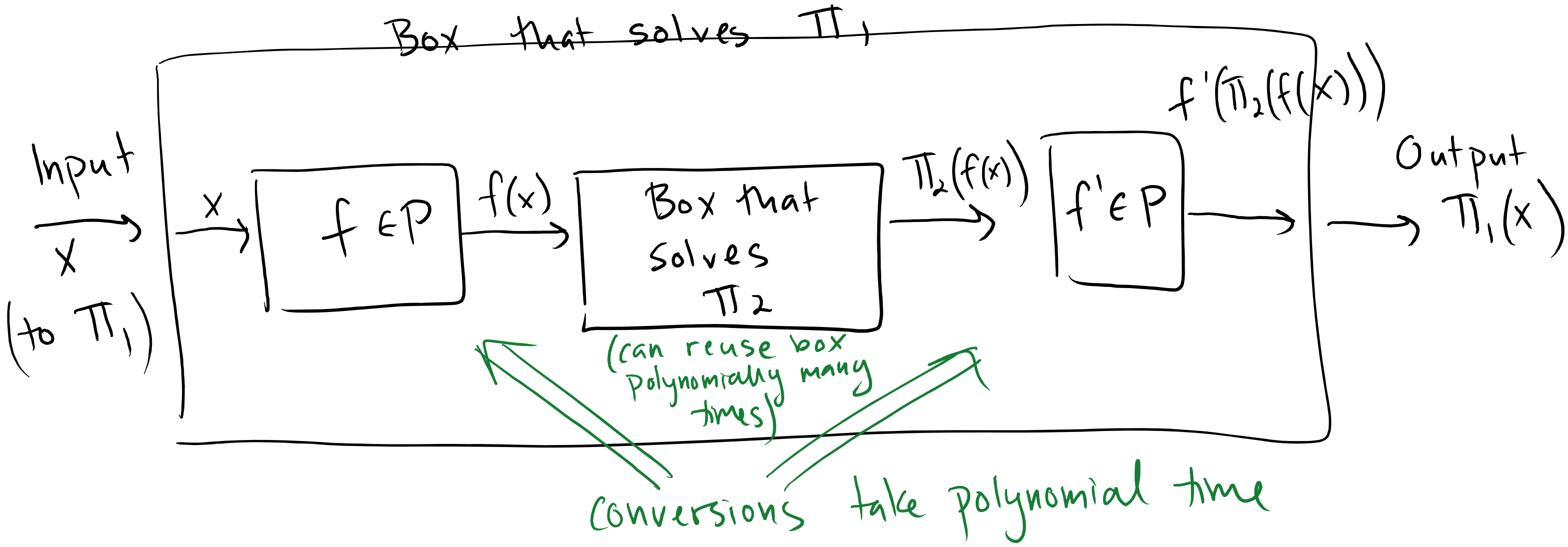
- TSP is hardest problem in NP

What does this mean?

Need concept of reduction

Polynomial Reduction

Problem Π_1 reduces to Π_2 if



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Q: If $\Pi_2 \in P$ and Π_1 reduces to Π_2 then

A) $\Pi_1 \in P$

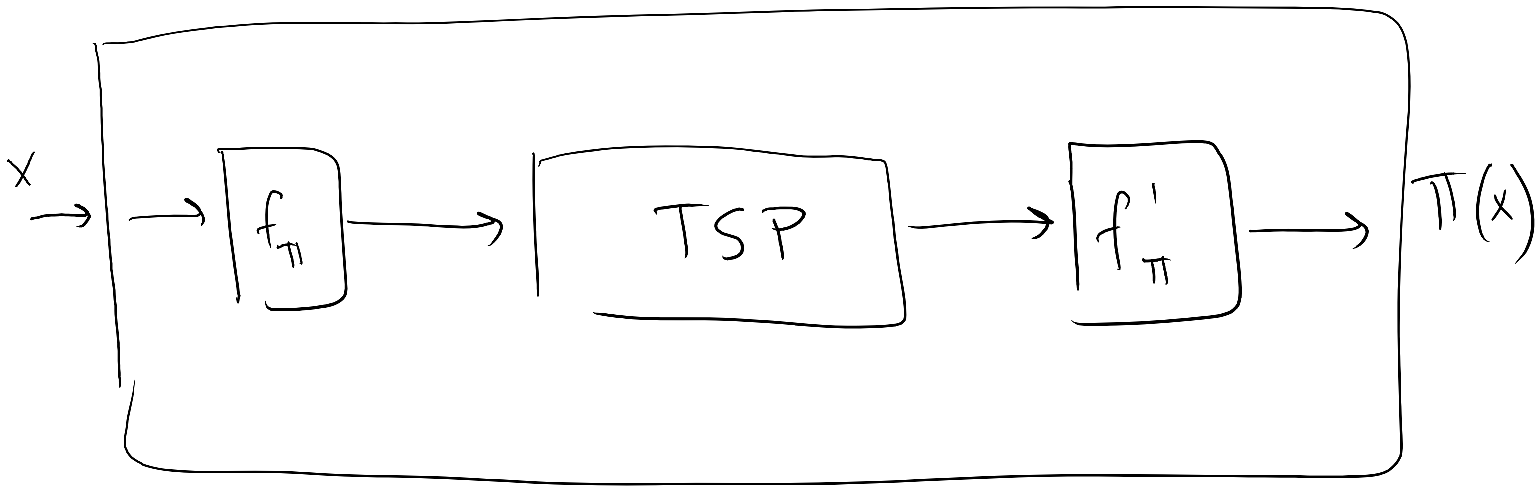
B) $\Pi_1 \notin NP$

C) We don't have enough information

$\Pi_1 \in P$ because need to run Π_2 poly # of times, takes poly time, and do poly time other steps $\Rightarrow$ poly time.

Π_1 reduces to Π_2 implies Π_2 is harder than Π_1 (if can do Π_2, can do Π_1)

All $\Pi \in NP$



- TSP is <u>NP-Hard</u> (every problem in NP reduces to it)
- Also <u>TSP∈NP</u>

$\Downarrow$ TSP is NP-Complete

$$\boxed{\Pi \in NP \land \Pi \in NP\text{-Hard} = NP\text{-COMPLETE}}$$

Another NP-complete problem:

<u>3SAT</u>: Given CNF formula of $x_1, x_2, \dots x_n$ and negations

$\uparrow$ AND of ORs

where each clause has at most 3 terms, is there a satisfying assignment?

e.g. $(x_1 \vee x_3 \vee \bar{x}_4) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_5) \wedge (\bar{x}_3 \vee x_5)$

$x_3 = 1$
$x_5 = 1$
$x_2 = 0$

Most people believe NP-Hard problems take exponential time to solve

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Important Skill in algorithm design proving Π is NP-Hard

Why?

- Won't spend time trying to find efficient solution
- Look up existing alg for NP-hard problems
- Use special structure (average case might be easy)

Strategy: Reduce $\underbrace{\text{3SAT to } \Pi}$

$\Downarrow$

Π harder 3SAT, 3SAT harder than NP

$\Rightarrow \Pi$ is harder than NP

<u>Prove</u>: 3SAT reduces to HAMPATH

1) Create $f(x)$, $f'(\Pi_2(f(x)))$

2) Show takes poly time to do reduction

3) If 3SAT(x) has solution, HAMPATH(f(x)) has soln

4) If HAMPATH(f(x)) has soln, 3SAT(x) has soln

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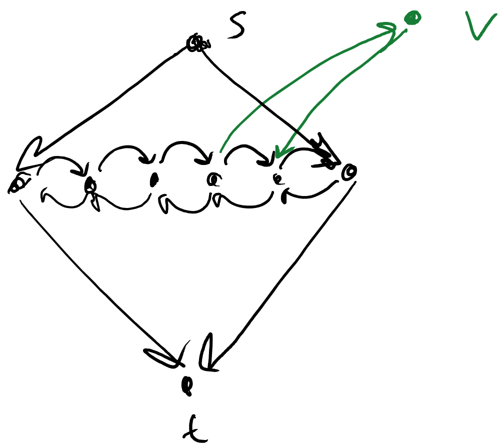

Q: How many Hamiltonian Paths are there from s to t without v / with v?

A) 1, 0

B) 1, 1

C) 1, 2

D) 2, 1 ⟸

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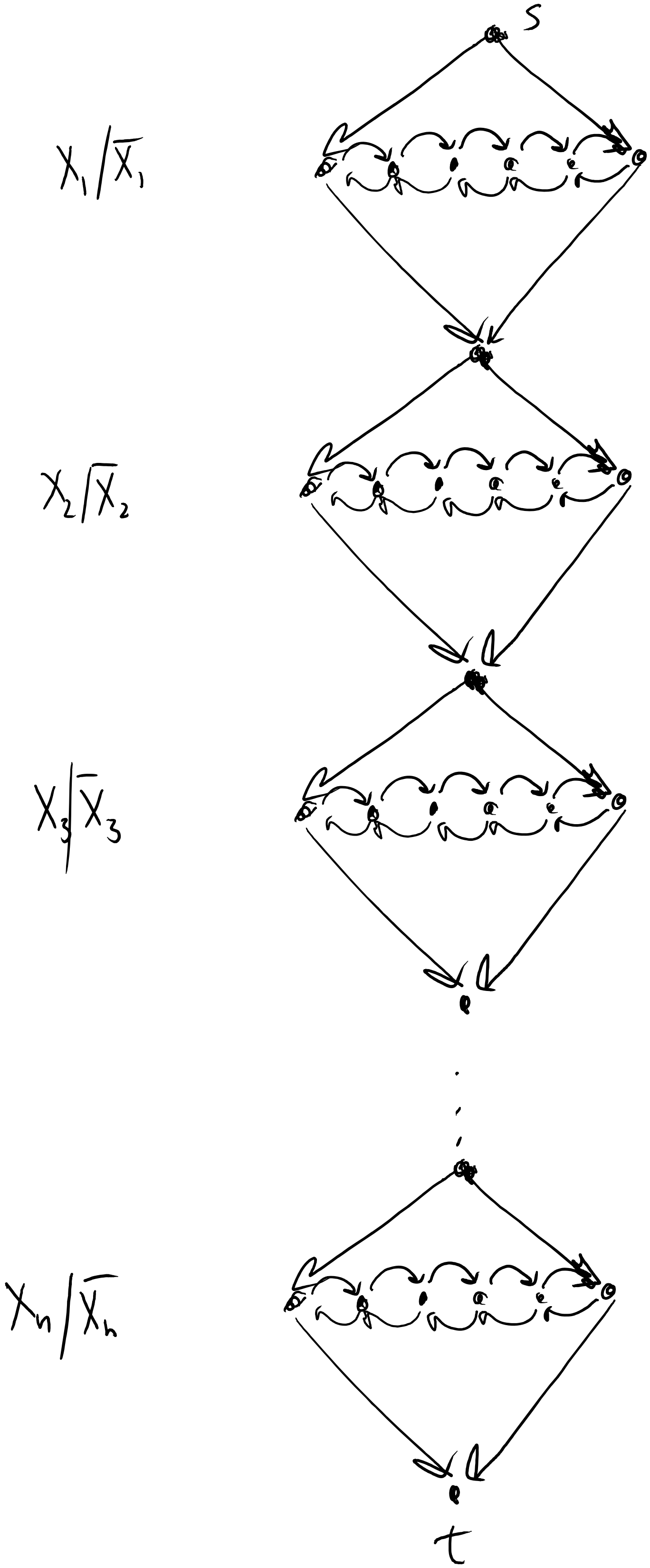
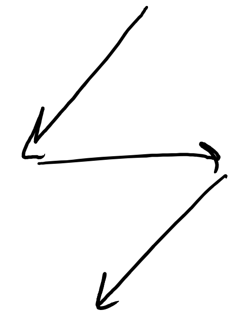
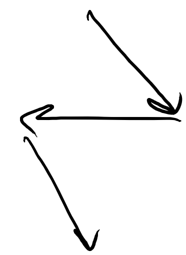


$x_i = 1$: zig



$x_i = 0$: zag



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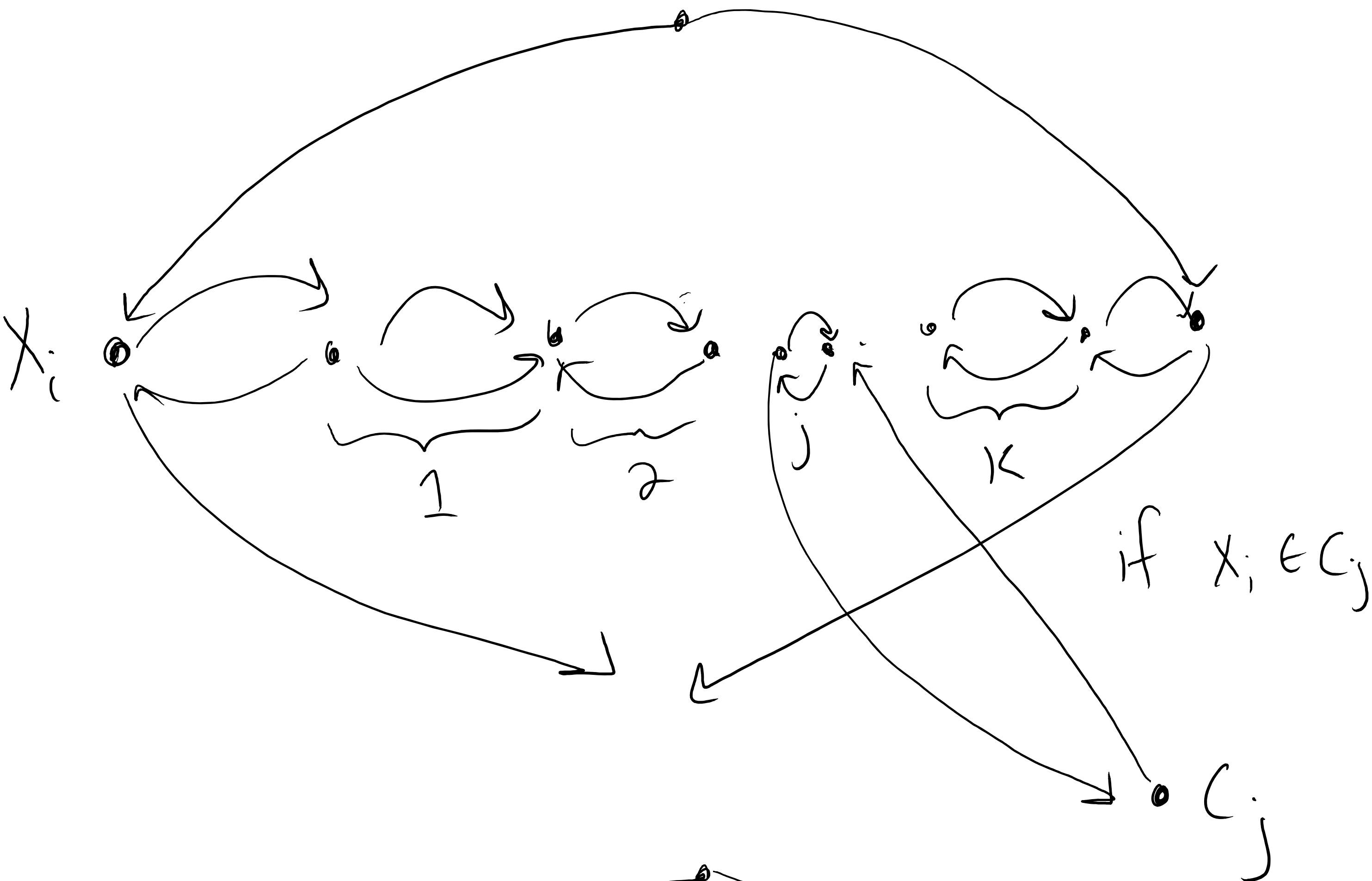

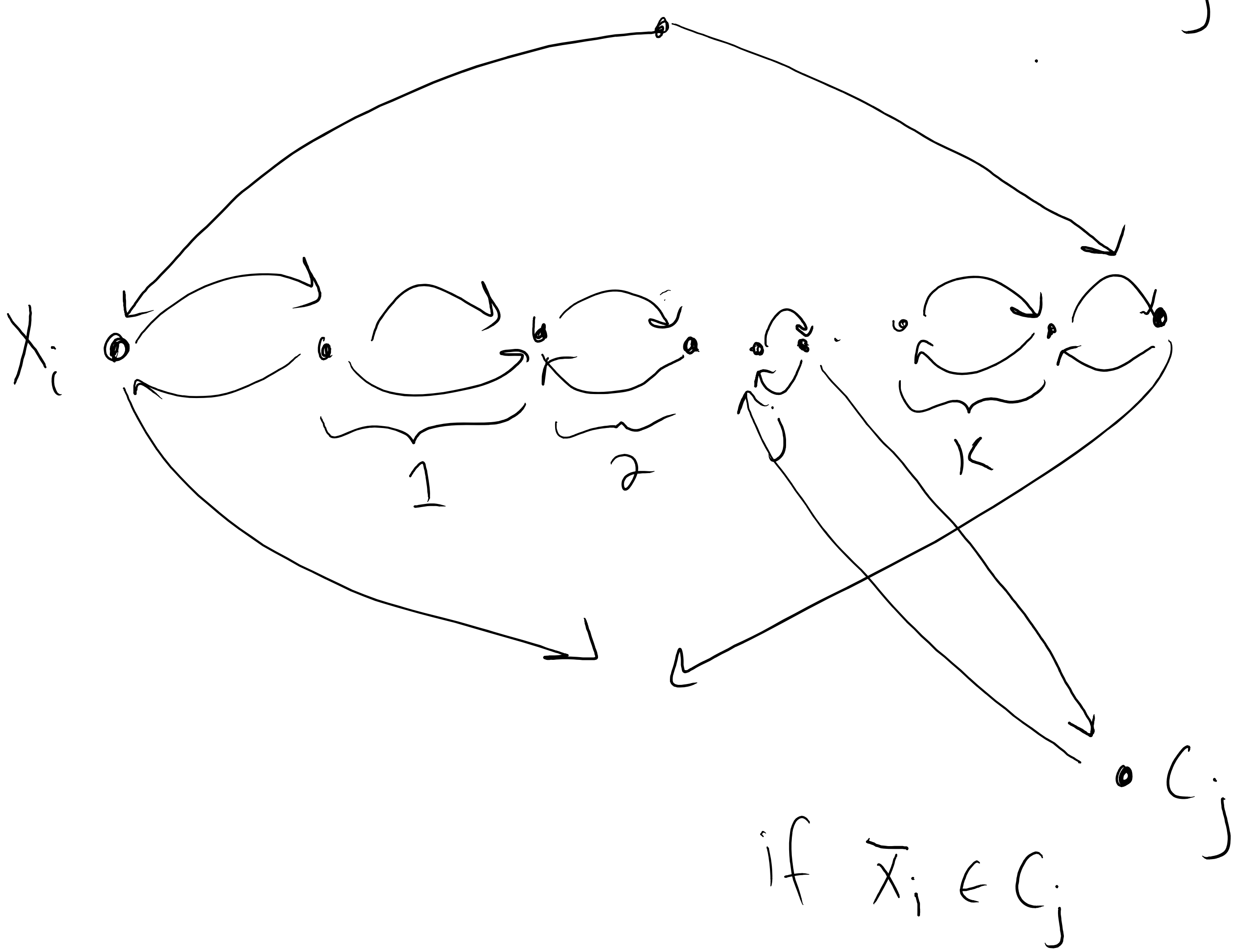

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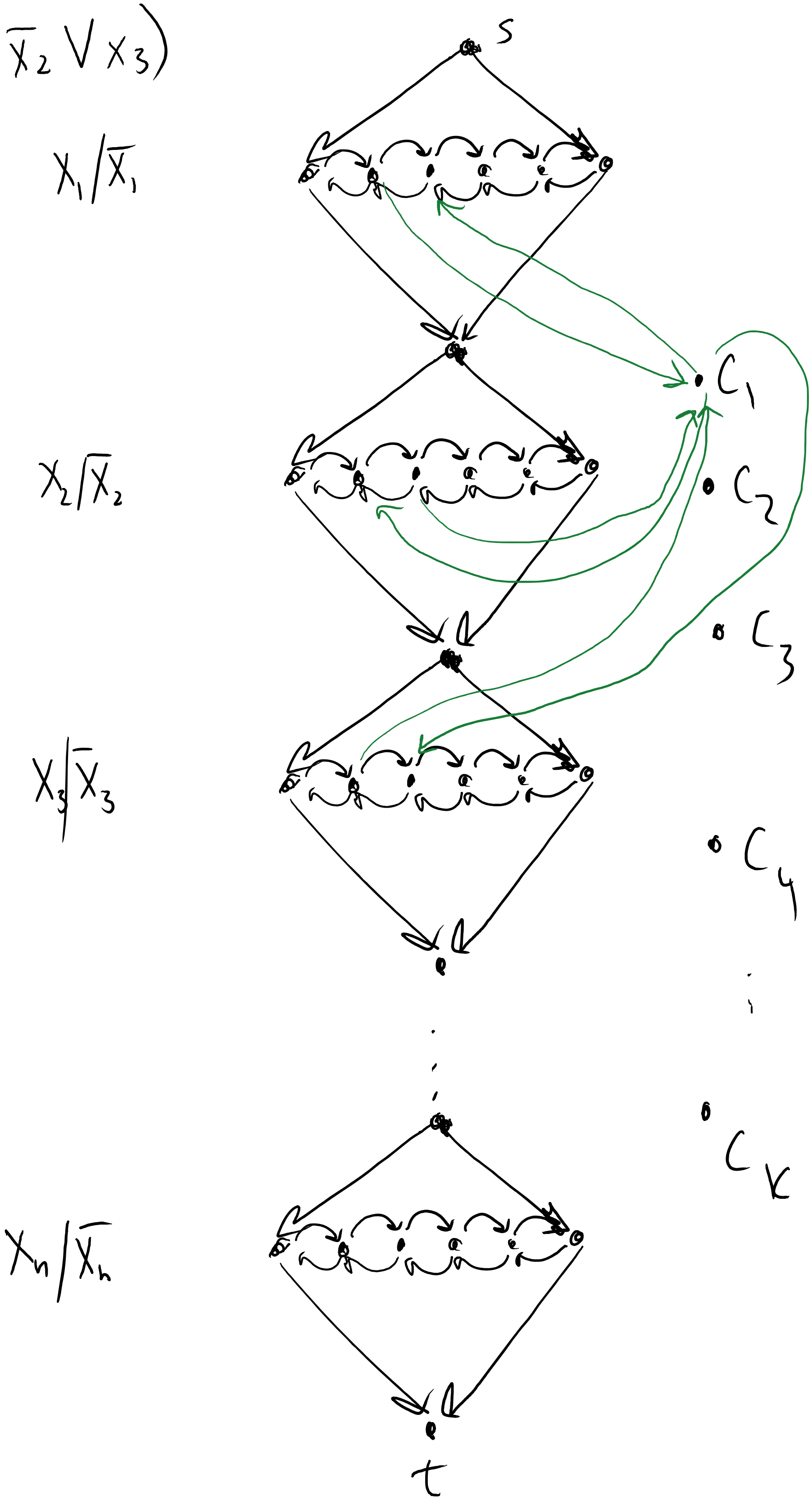
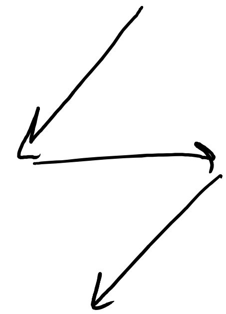
$(x_1 \vee \bar{x}_2 \vee x_3)$

Draw



$X_i = 1$: zig



$X_i = 0$: zag