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Goals

- Use permutations + combinations to solve problems
- Use Π, !, Σ, $\{\ \}^k$ notation appropriately

Quiz Topics → most recent PSet (or 2)

Permutation Warm-Up

Q: There are 10 singles left in Coffrin and you and 2 friends want to pick 3 of them. How many ways could you choose rooms.

A) 30 B) 300 C) 720 D) 1000

Answer: using product rule $10 \cdot 9 \cdot 8 = 720$

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List possibilities

Room of

Person 1	Person 2	Person 3

$(5, 7, 10)$

$(5, 10, 7)$

$(2, 3, 7)$

$\vdots$

3 permutation of $\{1, 2, 3, \ldots, 10\}$

Def: A <u>K-permutation of n elements</u> is

An ordering of a set of K elements where those K are chosen from n elements

ex: (a, c) is a 2-permutation of $\{a, b, c, d\}$.

$$\boxed{P: \mathbb{N} \times \mathbb{N} \to \mathbb{N}, \quad P(n,k) = \# \text{ of } k\text{-permutations of } n \text{ elements}}$$

Solution to Plicker question on previous page:

$$P(10,3)$$

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Q: Use product rule to find formula for $P(n,k)$.

A: $n \cdot (n-1)(n-2) \cdots (n-k+1)$

Notation: can write as $\prod_{i=n-k+1}^{n} i$.

Product Symbol / Summation Symbol

Given an ordered list of elements $(a_1, a_2, a_3, \dots)$

$$\prod_{i=j}^{k} = a_j \times a_{j+1} \times a_{j+2} \times \cdots \times a_k$$

$$\sum_{i=j}^{k} = a_j + a_{j+1} + a_{j+2} + \cdots + a_k$$

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Notation

$$n! = \prod_{i=1}^{n} i = 1 \cdot 2 \cdot 3 \cdot 4 \cdots n$$

Another way to write $P(n,k)$:

$$10 \cdot 9 \cdot 8 \left(\frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \right) = \frac{10!}{7!}$$

so $$P(n,k) = \frac{n!}{(n-k)!}$$

Q: There are 10 singles left in Coffin and you and 2 friends want to pick 3 of them.
Suppose you just want to pick 3 rooms now, and you'll figure out who will stay where later.
How many ways could you pick 3 rooms?

A) 30 B) 120 C) 240 D) 360

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We know 720 ways if care about order.

If care about order, thes are all different

$$\left\{ \begin{array}{l} (2,3,5), (2,5,3), (3,2,5), (3,5,2) \\ (5,2,3), (5,3,2) \end{array} \right.$$

My pick   Friend 1 pick   Friend 2 pick

But if don't care about order, these are all the same. $\{2,3,5\}$

$\Rightarrow$ Over counting by a factor of 6 for each set!

$$720/6 = 120$$

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Function

$C(n,r) = \binom{n}{r}$ = "n choose r" is the number of <u>sets</u> of r elements chosen from a set of n elements.

order doesn't matter

Fact: $P(n,r) = \binom{n}{r} \cdot P(r,r)$ **Why?**

$$\Rightarrow \binom{n}{r} = \frac{P(n,r)}{P(r,r)} = \frac{n!}{(n-r)!\left(\frac{r!}{1!}\right)} = \frac{n!}{(n-r)!\, r!}$$

Using the product rule.
The number of ways we can order r things chosen from among n things is equal to the number of subsets of r things, times the ways we can order each subset.

$$\boxed{\binom{n}{r} = \frac{n!}{(n-r)!\, r!}}$$