

CS200 - Midterm 1 Review Questions

1. [11 points] Prove using induction that program `ProductThing(n)` returns a number less than or equal to n^n for all $n \geq 1$.

Algorithm 1: `ProductThing(m)`

```

Input  :  $m \in \mathbb{Z}$  such that  $m \geq 1$ 
Output: Something.
// Base Case
1 if  $m == 1$  then
2   |   return 1;
3 else
4   |   // Recursive step
5   |   return ProductThing(m - 1)  $\times m$ 
6 end

```

2. Let S be the set of all people who have ever lived. Let $G(x, y)$ be the predicate, x is the grandmother of y , for $x, y \in S$. Let $C(x, y)$ be the predicate x and y are cousins for $x, y \in S$. Write the following statements and predicates using math:
- “All people have at least two grandmothers.”
 - “Every pair of cousins share a grandmother.”
 - “None of Person x ’s cousins are grandmothers ”
 - (Challenge) “All of Person x ’s children except one are childless.”
3. You meet a group of 50 orcs. You know orcs are either honest or corrupt. Suppose you know that at least one of the orcs is honest. You also know that given any two of the orcs, at least one is corrupt. Let G be the set of orcs, and $D(g)$ is the predicate “orc g is corrupt.”
- How many of the orcs are corrupt and how many are honest?
 - Express “At least one orc is honest” using math.
 - Express “Given any two orcs, at least one is corrupt” using math
4. (This is a little harder than I would ask on an exam, but good practice!) Prove that $1/1 + 1/4 + 1/9 + \dots + 1/n^2 \leq 2 - 1/n$ for all $n \geq 1$, .