

Do a detailed calculation of worst case # of operations

procedure *insertion sort*(a_1, a_2, \dots, a_n : real numbers with $n \geq 2$)

for $j := 2$ **to** n

$i := 1$

while $a_j > a_i$

$i := i + 1$

$m := a_j$

for $k := 0$ **to** $j - i - 1$

$a_{j-k} := a_{j-k-1}$

$a_i := m$

{ a_1, \dots, a_n is in increasing order}

What is the runtime?

$$\begin{aligned}\# \text{ of operations} &= \sum_{j=2}^n [\text{work done inside loop}] \\ &= \sum_{j=2}^n \left[1 + \sum_{i=1}^j 1 + \sum_{k=0}^{j-2} 1 \right] \\ &= \sum_{j=2}^n [1 + j + j - 1] = \sum_{j=2}^n [2j] = \frac{2(n+1)n}{2} - 2 = O(n^2)\end{aligned}$$