CS200 - Problem Set 12

1. Consider rolling 5 dice. Let $X_{i,j}$ be an indicator random variable that takes value 1 if the ith di has outcome j (and takes value 0 otherwise).

   (a) [**3 points**] What is the sample space? What is its size?

   (b) [**3 points**] Let X be the random variable that is the sum of all of the values shown on the dice. If the outcome of the rolls is $s = (4, 2, 4, 5, 1)$, what is $X(s)$? What is $X_{3,4}(s)$? What is $X_{4,3}(s)$?

   (c) [**3 points**] Write X in terms of a *weighted* sum of the variables $X_{i,j}$.

   (d) [**3 points**] What is $\mathbb{E}[X_{i,j}]$?

   (e) [**3 points**] Use linearity of expectation to determine the average value of the sum of all values shown on the dice.

2. Suppose a group of n people each order a different flavor of ice cream at an ice cream shop. Suppose the server didn't keep track of who ordered which flavor, and just handed the ice cream out randomly.

   (a) **3 points** Let X a random variable that is the number of people who got handed the correct flavor. Let X_i be the indicator random variable that takes value 1 if person i gets the correct flavor (and 0 otherwise.) Write X in terms of a sum of the X_i.

   (b) **6 points** Use linearity of expectation to determine the average number of people who get the correct flavor.

3. [**6 points**] Let $A = \mathbb{N}$, and $B = \{1, 2\} \times \mathbb{N}$. Show $|A| = |B|$.

4. [**11 points**] Let S be the the set of functions from $\mathbb{N}$ to $\{0, 1\}$. Prove S is uncountably infinite.

5. How long did you spend on this homework?