

# Inductive Proof Recipe:

- Set-up (need a predicate  $P(n)$ )
- Base Case
- Inductive Case (assume  $P(k)$ )
- Conclusion

Prove:  $2^n - 1 \leq 3^n$  for all  $n \geq 0$ .

## Set-up and Base Case

- Let  $P(n)$  be the predicate that  $2^n - 1 \leq 3^n$ . We will prove via induction that  $P(n)$  is true for all  $n \geq 0$ .
- Base case:  $P(0)$  is true because  $2^0 - 1 = 0$  while  $3^0 = 1$ , so  $2^0 - 1 \leq 3^0$ .

# Inductive Step

Inductive step: Let  $k \geq 0$ . We assume for induction that  $P(k)$  is true. This means we assume that

$$2^k - 1 \leq 3^k.$$

Multiplying both sides by 2 and then adding 1, we get

$$2^{k+1} - 1 \leq 2 \times 3^k + 1.$$

Now since  $k \geq 0$ , then  $1 \leq 3^k$ , so

$$2^{k+1} - 1 \leq 2 \times 3^k + 3^k = 3^k(2 + 1) = 3^{k+1}.$$

Therefore  $P(k + 1)$  is true.

# Conclusion

- Therefore, by induction on  $n$ ,  $P(n)$  is true for all  $n \geq 0$ .

# Recursive Algorithm Correctness

- Prove the following algorithm is correct

ReverseString( $s$ )

**Input** : String  $s$

**Output:** A string whose characters are the reverse  
of  $s$

// Base Case

1 **if**  $length(s) == 1$  **then**

2 | return  $s$ ;

3 **end**

// Recursive step

4 return  $s[end] + \text{ReverseString}(s[1 : end - 1])$ ;

## Set-up and Base Case

- Let  $P(n)$  be the predicate that ReverseString correctly reverses any string of length  $n$ . We will prove via induction that  $P(n)$  is true for all  $n \geq 1$ .
- Base case:  $P(1)$  is true because for strings of length 1, the reverse of the string is the same as the string, so the algorithm should just return the string. This is indeed what the algorithm does in lines 1 – 2.

# Inductive Step

- Inductive step: Let  $k \geq 1$ . We will prove  $P(k + 1)$ . Consider when the input  $s$  is a string of size  $k + 1$ . Since  $k \geq 1$ , then  $k + 1 \geq 2$ , so the algorithm goes to the recursive step. By our inductive assumption since  $s[1: \text{end} - 1]$  is a string of length  $k$ , `ReverseString(s[1: end - 1])` works correctly and returns the reverse of the first  $k$  characters of  $s$ . If we put the last character of  $s$  at the beginning of the reverse of the first  $k$  characters, we get the full reverse of  $s$ . This is what line 4 does, so  $P(k + 1)$  is true.

# Conclusion

- Therefore, by induction...