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Goals:

- Prove iff proofs
- Proof by example, counterexample
- Proof by cases

- Problem 1 (later today)
- Problem 2 (now) Statement is like a game. Opponent is $\forall$ You are $\exists$
- Problem 3 (see 2/25 slides, review, rest of semester)
- Texts / Office Hours...

Iff Proofs

Iff = "if and only if"

Use: $P \leftrightarrow Q$

$$(P \to Q) \land (Q \to P) \equiv P \leftrightarrow Q$$

(use truth table proof)

Structure

¶ For the forward direction, [Proof of $P \to Q$]

¶ For the backwards direction, [Proof of $Q \to P$]

Could be direct or contrapositive

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Proof by Example:

Q: Which could be proved using an example?

A) $\forall x \in S, P(x)$

B) $\forall x \in S, \neg P(x)$

C) $\neg \exists x \in S : P(x)$

D) $\neg \forall x \in S, P(x)$

Structure:

- Prove there exists...

  We give an example

- Prove not all ....

  We give a counterexample

ex: Prove: not all students in this class were born in the same month

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Proof by Example:

Q: Which could be proved using an example?

A) $\forall x \in S, P(x)$

B): $\forall x \in S, \neg P(x)$

de Morgan

C): $\neg \exists x \in S: P(x) \equiv \forall x \in S, \neg P(x)$

To show its true for all of S, a single example is not enough

$\boxed{\text{D)}}$ $\neg \forall x \in S, P(x) \equiv \exists x \in S: \neg P(x)$ $\Leftarrow$ To show there exists, a single example is enough

Structure:

- Prove there exists...

  We give an example

- Prove not all ....

  We give a counterexample

WARNING: If try to use proof by example to do a "for all" proof, you will be VERY wrong.

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Proof By Cases

We've already seen: When you proved

$(P \to Q) \vee (Q \to R) \equiv S$ is true.

Pf

We will prove S is true for any value of Q. There are two cases: Q is true or Q is false.

Case 1: If Q is true ..... therefore S is true

Case 2: If Q is false ... Therefore S is true.

From Pset:

$F \vee p \equiv ?$

2 cases:

p true → $F \vee T = T$

p false $F \vee F = F$

In either case, $F \vee p \equiv p$

Q: What is $p \to F$?

A) F B) T C) p D) $\neg p$

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Q: What is $p \to F$?

A) F   B) T   C) p $\boxed{D)\ \neg p}$

$p = T \Rightarrow T \to F \equiv F$

$p = F \Rightarrow F \to F \equiv T$

ex:

Let $P(n)$ be the predicate n cents of postage can be made from 5¢ and 8¢ stamps. We will prove $P(n)$ is true for all $n \geq 28$.

Base case:

Inductive step: let $k \geq 28$. We assume $P(k)$ is true. Let m be the number of 5 cent stamps used to create k. There are two cases: $m \geq 3$ or $m < 3$. If $m \geq 3$, we can remove 3 5¢ stamps and add 2 8¢ stamps to get $k+1$¢. If $m < 3$ then at most 10¢ out of k is created using 5¢ stamps. Because $k \geq 28$, at least 18¢ must be made from 8¢ stamps. This means there are at least 3 8¢ stamps used to make k¢. But then in this case we can remove 3 8¢ stamps and add 5 5¢ stamps to get $k+1$¢.

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Goals

- Describe & write proof by contradiction
- Describe & write proof by strong induction

Proof by Contradiction

Use: any statement P

Proof needs to do two things

$$\left.\begin{array}{ll} \text{① (Direct)} & \neg P \rightarrow Q \\ \text{② (Direct)} & \neg P \rightarrow \neg Q \\ \hline & \therefore P \end{array}\right\} \text{Most common}$$

Structure

For contradiction assume $\neg P$

Explain explain... Q

Explain explain... $\neg Q$, a contradiction.

Thus, P is true

When start, don't know what Q is... you need to keep your eye out for what might be the contradiction.