Math Foundations of Computer Science

Inductive Proof Recipe:

- Let $P(n)$ be the predicate ____________. We will prove, using induction on n, that $P(n)$ is true for all $n \geq$ _
- Base Case: $P(\_)$ is true because ____________
- Inductive Case: Let $k \geq$ _. Assume, for induction, that $P(k)$ is true. Then ____[a bunch of explanation and math here]____. Thus, $P(k+1)$ is true.
- Therefore, by induction, $P(n)$ is true for all $n \geq$ _.

Prove: $2^n + 1 \leq 3^n$ for all integers $n \geq 1$.

Prove: Sum of first n odd numbers is n^2.

Set-up

- Let $P(n)$ be the predicate $2^n + 1 \leq 3^n$. We will prove via induction that $P(n)$ is true for all $n \geq 1$.

Base Case

- $P(1)$ is true because $2^1 + 1 = 3^1$.

Inductive Case

Let $k \geq 1$. Assume for induction that $P(k)$ is true. This means
$$2^k + 1 \leq 3^k.$$

Multiplying both sides by 2 and then subtracting 1, we get
$$2^{k+1} + 1 \leq 2 \times 3^k - 1.$$

Now, $2 \times 3^k = 2 \times 3^k + 3^k - 3^k = 3^{k+1} - 3^k$. Plugging in:
$$2^{k+1} + 1 \leq 3^{k+1} - 3^k - 1.$$

Now since $-3^k - 1 \leq 0$, we finally have
$$2^{k+1} + 1 \leq 3^{k+1}.$$

Thus $P(k+1)$ is true.

Conclusion

- Therefore, by induction on n, $P(n)$ is true for all $n \geq 1$.