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Goals

- Describe functions and their properties
- Analyze complexity of simple algorithms
- Apply Big-O

Announcements

- Tea
- PA2 available

Due before Wed Class

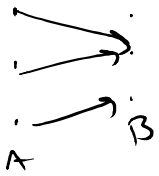
- Survey
- Pre Quiz

Due Thurs:

- Self Grade / Reflection

With group, review notes on functions from last week, come up with at least one question about functions.

To describe a function (formally):

- $f: A \to B$ (define domain and codomain)
- Describe mapping. For example
  - $f(x) = \ldots$
  - 

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If clear from context, we will sometimes not explicitly state domain & codomain.

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Intro to Algorithm Complexity

Important function: worst case time complexity of an algorithm in C.S.

$T: D \to \mathbb{N}$, for $D \subseteq \mathbb{N}$

- D: input size
- T: # of operations performed by algorithm in worst case.

unless parallel computing, this tells you the time the computer will take to run the algorithm. Just multiply by time to do 1 operation

Linear search

- Input: $(a_1, a_2, \ldots a_n)$, x ⟸ Input size is n
- Output: j if $a_j = x$, 0 otherwise

```
1) i = 1
2) while (i ≤ n and x ≠ a_i)
3)       i = i+1
4) if i ≤ n:
       return i
5) else:
       return 0
```

Q: What is $T(n)$ for linear search? (Hint: n is not correct)

Report by By group?

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Issues:

- too fine-grained / detailed
  - different computers might do operations differently
  - when n gets large, don't care about 100000 vs 100001
- too difficult to count every operation

Big-O to Rescue!

↑ special notation to describe how functions grow

def: Let $f, g: \mathbb{Z} \to \mathbb{R}^+$ or $f, g: \mathbb{Z} \to \mathbb{N}$,

Then $f(x)$ is $O(g(x))$ if $\exists\, k, C \in \mathbb{R} : \forall\, x \geq k$,

$$f(x) \leq C\, g(x).$$

"f of x is big-oh of g of x"

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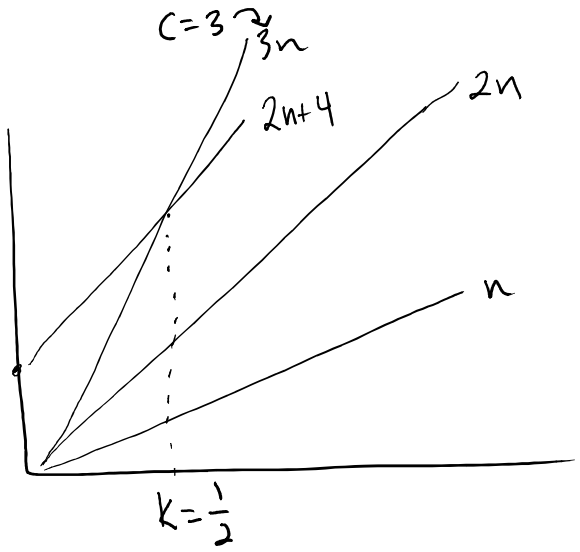
ex: $2n+4$ is $O(n) \equiv \exists\, k, C \in \mathbb{R}: 2n+4 \leq Cn \quad \forall\, n \geq k$



(often use x, n as function inputs)

$\Longleftarrow$ Not a proof.

Proof:

- For $n \geq 1$, we have $4n \geq 4$. (Multiply both sides by 4). Thus for $n \geq 1$, $2n+4 \leq 2n+4n = 6n$, so $2n+4 = O(n)$ with $k=1$, $C=6$.

- For $n \geq 4$, we have $2n+4 \leq 2n+n = 3n$, so $2n+4 = O(n)$ with $k=4$, $C=3$.

Infinitely many combos of k, C work. To prove, you need to find <u>ONE</u> combo.

General trick: try to get $f(x)$ to look like $g(x)$ by turning bad terms into good terms.

Above $4 \to 4n$ or $4 \to n$

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Q: Prove other functions for linear search # of operations is $O(n)$

Starting to see why big-O is good for algorithm time complexity:

- Small differences in how you calculate don't matter
- Not too fine grained

However big-O is only upper bound:

ex: $7x+1$ is $O(x^2)$

Pf: We have $7x+1 \leq 7x+x$ for all $x \geq 1$

Then $7x+x = 8x \leq x^2$ for all $x \geq 8$

Thus with $k=8$, $C=1$, $7x+1 = O(x^2)$

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Q: Prove $10x^2$ is not $O(x)$. This means There do not exist constants k, C, such that $10x^2 < Cx$ $\forall\ x \geq k$.

Pf: For contradiction, assume k, C exist. Then $\forall\ x \geq k$, we have

$$10x^2 \leq Cx$$

When $x > 0$, we have $x \leq \frac{C}{10}$. Thus, this inequality holds only when $0 < x \leq \frac{C}{10}$, which contradicts that it should hold for all $x \geq k$.