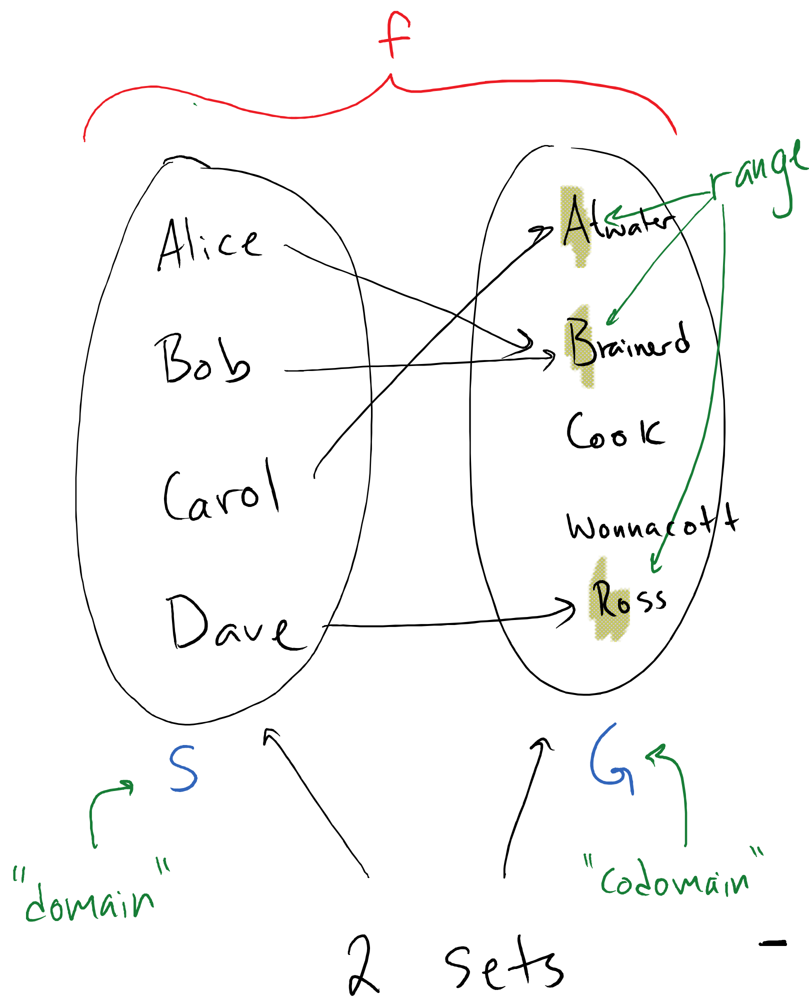
S.KIMMEL

Functions

All this notation is important because when we write about functions we need to have accurate words

ex: Commons affiliation



You give the function a student name as input, it gives a grade as output

We write:

- $f: S \to G$

means "f is a function from domain S to codomain G"

- $f(\text{carol}) = \text{Atwater}$

- Atwater is "image" of carol
- Carol is "preimage" of Atwater

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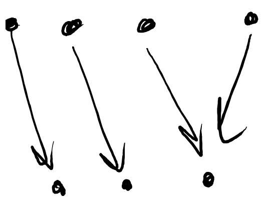
3 important properties

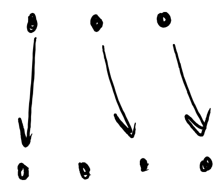
Surjection "Onto"



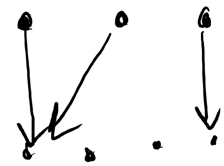
Surjective

Not Surjective

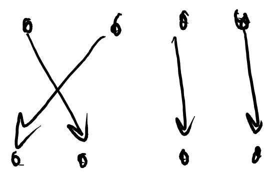
"One-to-One" Injection



Injective



Not injective



Injective & Surjective
= **Bijective**

Name ______________________________

CS200 - Functions Worksheet

1. Let $f : \{1, 2, 3\} \rightarrow \{1, 2, 3\}$. Which of the following is an incorrect representation of f?

A

$1 \rightarrow 3$

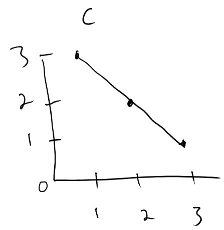
$2 \rightarrow 2$

$3 \rightarrow 1$

B

$f(x) = -x + 4$

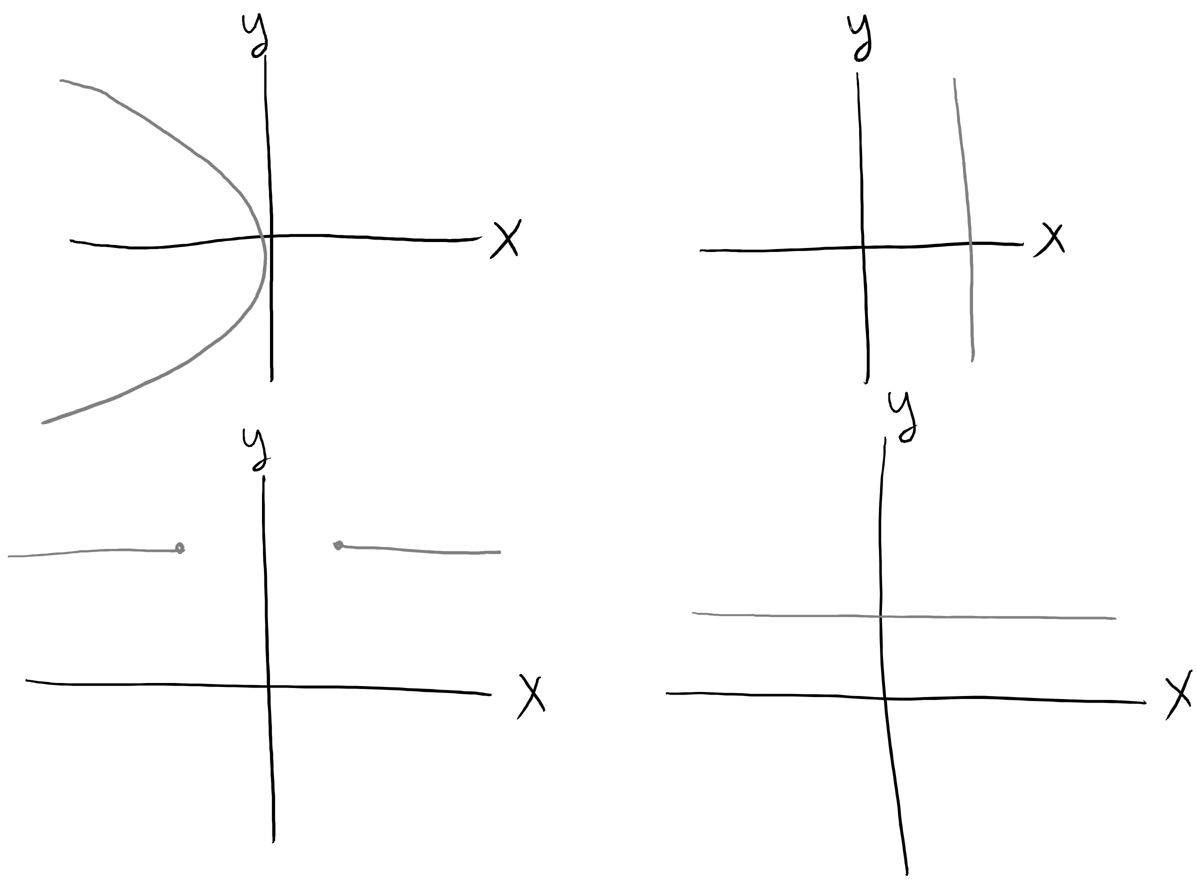
C



D

X	1	2	3
f(x)	3	2	1

2. Consider a function $f : \mathbb{R} \rightarrow \mathbb{R}$. If $y = f(x)$, we can depict f as a graph. Which of the following graphs (if any) could depict f?



3. Think of a real world function that is

- surjective

- injective
- bijective
- neither surjective nor injective

4. When you write a function in python or a method in java, what are typical domains and co-domains?

5. A relation R from the set A to B is a subset of $A \times B$. That is, $R \subset A \times B$. Explain why we can think of every function as a relation. Prove true or prove false that every relation represents a function.

6. English to Math, and just plain English: explain in words (using the new vocabulary you've learned) what each of the following means, and then express each using only mathematical notation.

   (a) A function $f : S \to G$ is surjective $\equiv$

   (b) A function $f : S \to G$ is injective $\equiv$

7. A function is strictly increasing if $f(x) < f(y)$ whenever $x < y$. A function is increasing if $f(x) \leq f(y)$ whenever $x < y$.

   (a) Prove that a if $f : \mathbb{R} \to \mathbb{R}$ is strictly increasing, then it is injective.

   (b) Prove that there exists an increasing function that is not injective.

8. Let $f : \mathbb{Z} \to \mathbb{N}$ be the function $f(x) = x^2$. For each of the following, please give an explanation.

   (a) [**6 points**] Is f surjective?

   (b) [**6 points**] Is f injective?