CS200 - Problem Set 8

Due: Monday, April 23 to Canvas before class

1. Graph representations:

   (a) **[9 points total]** (See grading rubric for pseudocode grading scheme.) Fill in the pseudocode for the following algorithms. You can assume that the graph is not directed, has no self-loops, and all edges have weight 1.

   Algorithm 1: `DegAdMat`(A, v)

   Input : Adjacency Matrix A of a graph $G = (V, E)$. A vertex $v \in V$.
   Output: Degree of the vertex v.

   Algorithm 2: `DegAdList`(A, v)

   Input : Adjacency List A of a graph $G = (V, E)$. A vertex $v \in V$.
   Output: Degree of the vertex v.

   Algorithm 3: `EdgeAdMat`(A, u, v)

   Input : Adjacency Matrix A of a graph $G = (V, E)$. Vertices $u, v \in V$.
   Output: 1 if edge between u and v, zero otherwise.

   Algorithm 4: `EdgeAdList`(A, u, v)

   Input : Adjacency List A of a graph $G = (V, E)$. Vertices $u, v \in V$.
   Output: 1 if edge between u and v, zero otherwise.

   (b) **[6 points]** Give an asymptotic, worst-case time complexity for each of your algorithms. (Whenever you see asymptotic, that means "big-O.") Explain.

2. **[6 points each]** Consider the following relations on the set of all humans. For each relation, explain whether or not the relation is reflexive, symmetric, and transitive. If it is an equivalence relation, what are the equivalence classes corresponding to the relation?

(a) $(a, b) \in R$ if and only if a and b have a common grandparent.

(b) $(a, b) \in R$ if and only if a and b have the same first name.

(c) $(a, b) \in R$ if and only if a is the same height or taller than b.

3. We studied the following algorithm in CS302.

Algorithm 5: `dynamic`(n)

Input: An $n \times 3$ array L containing natural numbers.
$A, B \in \mathbb{N}$, a rectangular array Q of size $A \times B$ with all 0's

```
1  for k=1 to A do
2      for j=1 to B do
3          for q=1 to k-1 do
4              if Q[k,j] < Q[q,j] + Q[k-q,j] then
5                  Q[k,j] := Q[q,j] + Q[k-q,j];
6              end
7          end
8          for r=1 to j-1 do
9              if Q[k,j] < Q[k,r] + Q[k,j-r] then
10                 Q[k,j] := Q[q,j] + Q[k-q,j];
11             end
12         end
13         for i=1 to n do
14             if (k = L[i,1] and j = L[i,2]) or
                (k = L[i,2] and j = L[i,1]) then
15                 if Q[k,j] < L[i,3] then
16                     Q[k,j] := L[i,3];
17                 end
18             end
19         end
20     end
21 end
22 return Q[A,B];
```

(a) **[3 points]** Write an expression using summation notation for the number operations used. (Do not analyze the expression)

(b) **[6 points]** Analyze the expression from part a by taking the sums, and then create an asymptotic bound for this expression. (Your bound should be a sum of terms of the form $n^x A^y B^z$ for x, y, z non-negative integers.)

(c) **[6 points]** Explain how you can bound the asymptotic runtime without using summation notation, but instead using a worst-case analysis.

4. **[6 points each]** For many card games, it is helpful to know the probability of certain hands occuring. In a standard deck, there are 52 cards: there are 13 different kinds of cards (Aces,

2s, 3s,...,10s, Jacks, Queens, Kings) and each kind appears 4 times, one for each of four suits (hearts, spades, diamonds, clubs).

(a) If you are dealt 5 cards, and all possible hands of cards are equally likely, what is the probability that you get 4 of the same kind (for example, 4 Jacks)? (The order in which you receive cards doesn't matter.)

(b) If you are dealt 5 cards, and all possible hands of cards are equally likely, what is the probability that you get a full house, where a full house is 3 cards of one kind, and 2 cards of a different kind? (The order in which you receive cards doesn't matter.)

5. **[6 points]** Suppose you create a graph on n vertices by looking at each pair of vertices, and then choosing to put an edge there with probability 2/3. What is the sample space of this problem? What is the size of the sample space?

6. How long did you spend on this homework?