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Announcements: Quiz Friday, Plickers, Questionnaire

Motivating Proofs

Q: When you write a program, how do you tell if it works correctly?

- Try examples
- Trace variable values
- Use debugging tools
- Think logically
- See if got an "A"

Better approach: **Proof**: formal method of arguing a statement is true

"My program outputs correct value"

Outline of Course

1. Writing Proofs
2. Important Math for C.S. (and life!)
    - counting
    - growth of functions
    - graphs
    - probability

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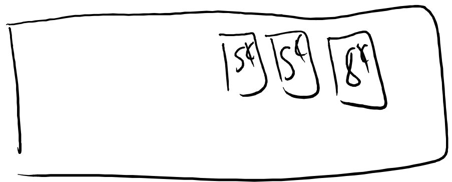
Induction

Suppose you have unlimited 5¢ stamps and 8¢ stamps.
What postage values can you create?



18¢ ✓

What about 4¢? No!

What about 28¢? Yes!

$\boxed{5}\boxed{5}\boxed{5}\boxed{5}\boxed{8}$

What about 85,694¢? ???

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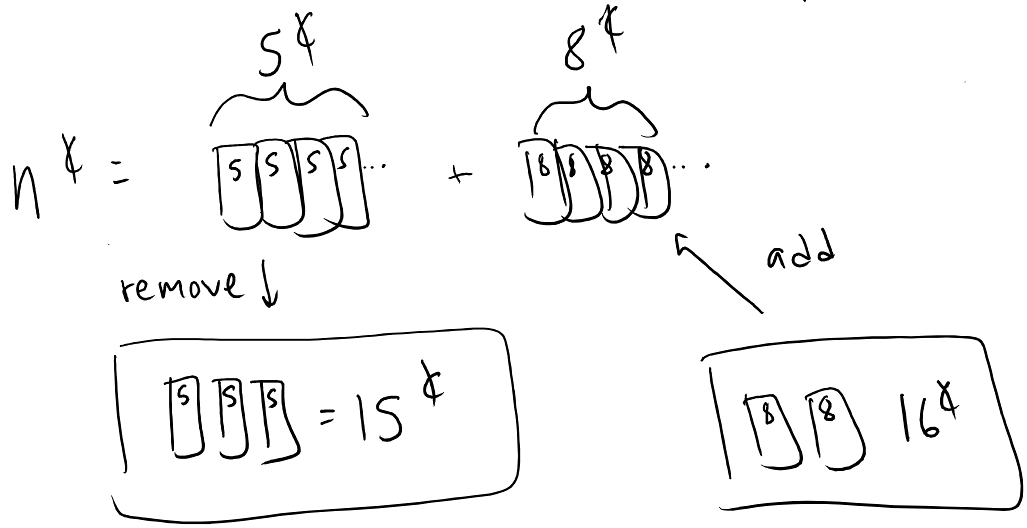
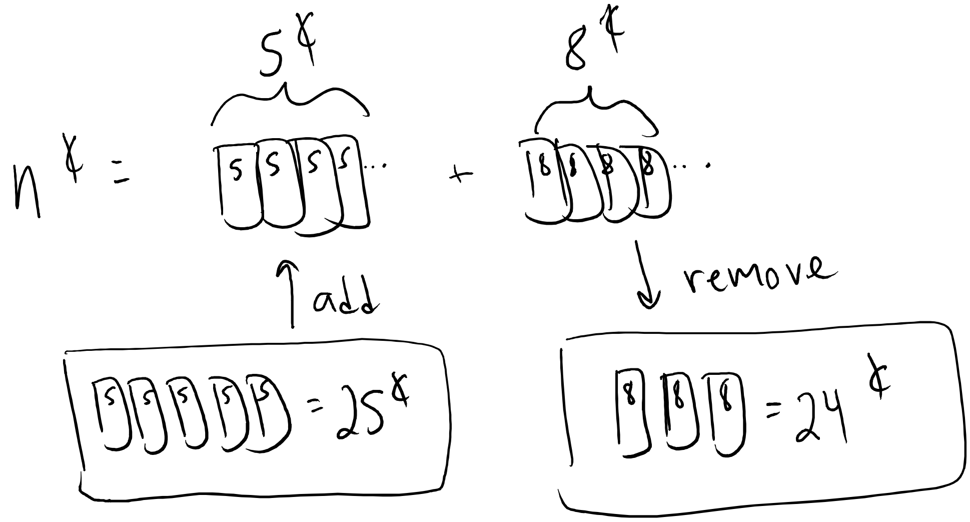
Induction: use old solution to get new solution

Suppose



$n¢ \to (n+1)¢$

Suppose



$n¢ \to (n+1)¢$

Consequence: If can create $n¢$ with at least 3 $\boxed{5}$ or at least 3 $\boxed{8}$, can create $n+1$ ¢

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$28¢ = 4 \cdot \boxed{5} + 1 \cdot \boxed{8}$

$29¢ = 1 \cdot \boxed{5} + 3 \cdot \boxed{8}$

$30¢ = 6 \boxed{5}$

$31¢ = 3 \boxed{5} + 2 \cdot \boxed{8}$

$\vdots$

Q: If $85{,}693¢ =$

$5{,}761 \boxed{5} + 7111 \cdot \boxed{8}$

then can create

$85{,}694¢$ as

$\underline{5758} \boxed{5} + \underline{7113} \boxed{8}$

or

$\underline{5766} \boxed{5} + \underline{7108} \boxed{8}$

A) 5759 / 7114

B) 5764 / 7108

C) 5766 / 7108 ⟸

D) 5758 / 7113 ⟸

find first solution, &
the rest fall into place

⇓

* Any postage $\geq 28¢$ is possible

Start at $28¢ \to 29¢ \to 30¢ \ldots\ 85{,}693¢ \ldots$

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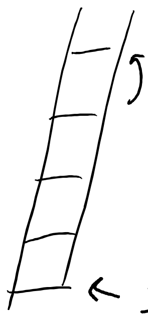
Principle of Induction: solution to smaller problem provides solution to larger problem

Stamps - need to have solution to n to get to n+1

Once you get 28¢ solution, we're good - always at least 3 5¢ or 8¢

Inductive Metaphor

Ladder



2. Show how to move from each rung to next

1. Show how to get on first rung (1st solution)

Formal Inductive Proof

Proofs have a unique style/language

- Essay vs. Texting vs News article vs. lab notebook

Different writing styles

This class → proof language.

Induction proof has a recipe, so easier style than other proofs.

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Inductive proof recipe:

like a function

(Set-Up)

Let P(n) be the predicate n¢ of postage can be formed from 5¢ and 8¢ stamps.

We will prove, using induction on n, that P(n) is true for all $n \geq$ 28.

(Base Case)

Base case: P(28) is true because ______

(Inductive Step)

base case #

Inductive case: Let $k \geq$ 28. Assume, for induction, that P(k) is true.

That means ______

So ______

Then ______

Plugging in ______

Each sentence logically follows from previous.

Thus P(k+1) is true

(Conclusion)

Therefore, by induction, P(n) is true for all $n \geq$ __ .

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Notice:

- Complete sentences or equations. Should be able to smoothly read aloud.

Q: Write inductive proof that $7^n - 1$ is a multiple of 6 for all integers $n \geq 0$.

(Hint: X is a multiple of 6 if $X = 6 \cdot m$ for some integer m.)

P Let $P(n)$ be the predicate $7^n - 1$ is a multiple of 6. We will prove via induction that $P(n)$ is true for all $n \geq 0$.

P Base Case: $P(0)$ is the statement $7^0 - 1$ is a multiple of 6.

$7^0 - 1 = 1 - 1 = 0 = 0 \cdot 6$, so the statement is true.

P Inductive Case: Let $k \geq 0$. Assume, for induction, that $P(k)$ is true. This means $7^k - 1 = 6 \cdot m$ for some integer m.

Multiplying both sides by 7, we get

$$7(7^k - 1) = 7^{k+1} - 7 = 7 \cdot 6m$$

Adding 6 to both sides, we get

$$7^{k+1} - 1 = 6(7m + 1).$$

Since $7m-1$ is an integer, $7^{k+1}-1$ is divisible by 6. Thus $P(k+1)$ is true.

$\P$ Therefore, by induction on n, $P(n)$ is true for all $n \geq 0$.