Goals

- Describe proof by contrapositive, iff proof, proof by cases, proof by example/counter example.
- Practice writing proofs

Announcements

- Question when doing your self-assessment…contact your assigned TA!

Proof By Cases

Use a proof by cases to show:

For any integer n, the number $n^3 - n$ is even.

If finish, please sit and work on:
If k is a multiple of 4, then $\exists n \in \mathbb{N}$:

$$k = 1 + (-1)^n(2n - 1)$$

Proof By Cases

There are two cases, n is even and n is odd. If n is even, then $\exists k \in \mathbb{Z}: 2k = n$. Then $n^3 - n = 8k^3 - 2k = 2(k^3 - k)$. Since $(k^3 - k) \in \mathbb{Z}$, $n^3 - n$ is even.

If n is odd, $\exists k \in \mathbb{Z}: 2k + 1 = n$. Then $n^3 - n = n(n^2 - 1) = (2k + 1)(4k^2 + 4k + 1 - 1) = 2(2k + 1)(2k^2 + 2k)$. Since $(2k + 1)(2k^2 + 2k)$ is an integer, $n^3 - n$ is even in this case, too.

Proof By Cases

If k is a multiple of 4, then $\exists n \in \mathbb{N}$:

$$k = 1 + (-1)^n(2n - 1)$$

Solution sketch:

2 cases:

- k is greater than or equal to 0, and then $n = k/2$
- $k < 0$, and then $n = -\frac{k}{2} + 1$

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