CS200 - Worksheet 2

Prove using induction that the program Sum(A) outputs the sum of an List A

Input : List A of integers
Output: Sum of the elements of A.

```
1 l=length(A);
  // Base Case
2 if l equals 1 then
3 |  return A[1];
4 else
  |  // Recursive step
5 |  return Sum(A[1 : l − 1]) + A[l];
  |  // A[1 : l − 1] is a list containing the first l − 1 elements of A.
6 end
```

Algorithm 1: Sum(A)

Solution Let $P(n)$ be the predicate that Sum(A) outputs the sum of the elements of A for any list of length n. We will prove $P(n)$ is true for all $n \geq 1$.

Base case: when $n = 1$, the list only has one element, the sum of all of the elements in the list is just the value of that element. When $n = 1$, the base case triggers in line 2 and we return the value of the one element of A, which is correct.

Inductive step: Let $k \geq 1$. We assume for induction that $P(k)$ is true. Let's analyze what happens when the input to Sum is a list with $k + 1$ elements. Since $k \geq 1$, $k + 1 \geq 2$, so the algorithm goes to the recursive step in line 5, and returns sum($A[1 : l - 1]) + A[l]$. Since $A[1 : l - 1]$ is a list with k elements, by inductive assumption, sum($A[1 : l - 1]$) correctly returns the sum of the k elements, which is the sum of the first k elements of A. But now the sum of all $k + 1$ elements of A is just the sum of the first k elements, plus the final element. This is precisely what line 5 returns, so the outcome is correct.

Therefore, by induction $P(n)$ is true for all $n \geq 1$.