CS200 - Worksheet 1

(Taken from *Discrete Mathematics, an Open Introduction* by Levin). Each of the following proofs proves the same statement, but using a different approach. Read carefully, and then discuss the language used in the proofs with your group. Particular English words are used to signal mathematical meaning to the reader. Try to pick out these words are figure out what they mean. What else do you notice about the style or language? If you finish discussing language, write using math the statement that these proofs are all proving.

1. Let a, b be integers, and suppose a and b are odd. That is, $a = 2k + 1$ and $b = 2m + 1$ for some integers k and m. Then

$$\begin{aligned} ab &= (2k + 1)(2m + 1) \\ &= 4km + 2k + 2m + 1 \\ &= 2(2km + k + m) + 1. \end{aligned} \tag{1}$$

   Therefore, ab is odd.

2. Let a, b be integers, and suppose that ab is even but a and b are both odd. Namely, $a = 2k+1$ and $b = 2j + 1$ for some integers k, and j. Then

$$\begin{aligned} ab &= (2k + 1)(2j + 1) \\ &= 4kj + 2k + 2j + 1 \\ &= 2(2kj + k + j) + 1. \end{aligned} \tag{2}$$

   But this means that ab is odd, which contradicts our premise. Thus a and b can not both be odd.

3. Let a, b be integers and assume ab is even. Namely, $ab = 2n$ for some integer n. Then there are two cases: a must be either even or odd. If it is odd, then $a = 2k + 1$ for some integer k. Then we have

$$\begin{aligned} 2n &= (2k + 1)b \\ &= 2kb + b. \end{aligned} \tag{3}$$

   Subtracting $2kb$ from both sides, we get

$$2(n - kb) = b. \tag{4}$$

   Therefore, b must be even. The other case is that a is even, so we find that either a or b is even.