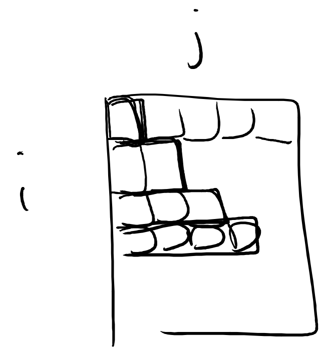
S.KIMMEL

Input: Adjacency Matrix A for $G=(V,E)$, G unweighted, undirected

Output: ??

1. $S=0$
2. for $i=1$ to $|V|$:
3. for $j=1$ to i:
4. $S = S + A[i,j]$
5. return S



Returns $|E|$

How many operations?

- Use Σ for loops
- Use 1 for $O(1)$ operations

some constant

$$\#\text{operations} = D + \sum_{i=1}^{|V|}\left[\text{work done inside } i^{th} \text{ loop iteration}\right]$$

line 1 & 5 line 2

some constant

$$= D + \sum_{i=1}^{|V|}\left[\sum_{j=1}^{i} K\right]$$

line 3 line 4

Write your expression from outer loops to inner loop

S. KIMMEL

Evaluate from the inside out:

$$\# \text{ operations} = D + \sum_{i=1}^{|V|} \left[\sum_{j=1}^{i} K \right]$$

$$= D + K \sum_{i=1}^{|V|} i$$

$$= D + K\left[1 + 2 + 3 + \dots + |V|\right]$$

$$= D + K(|V|+1)\frac{|V|}{2}$$

You proved when we did induction! "Arithmetic Series"

$$= O(|V|^2)$$

"Detailed Calculation"

"Rough Calculation"

Outer loop repeats $O(|V|)$ times
Inner loop repeats $O(|V|)$ times

$\Big\}$ $O(|V|^2)$ operations in worst case

S.KIMMEL

Summation Tricks

$$\sum_{i=2}^{n}(Ai+B) = \sum_{i=2}^{n}Ai + \sum_{i=2}^{n}B$$

$$= A\sum_{i=2}^{n}i + (n-1)B$$

$$\underbrace{2 + \underbrace{3 + \underbrace{4 \cdots + (n-2)}_{n+2} + (n-1)}_{n+2} + n}_{n+2}$$

How many pairs? $\frac{n-1}{2}$ ← # terms in series

Total: $\frac{(n-1)}{2} \cdot (n+2)$