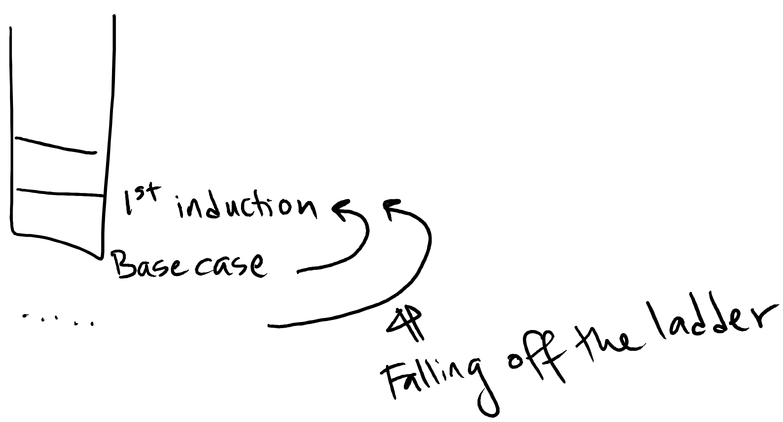
S.KIMMEL

Multiple Base Cases

- Start with one Base case
- Check if you fall off the bottom of ladder
    - ↳ Add base case



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$F(n)$
1. If $n \le 1$: return n
2. return $5 \cdot F(n-1) - 6 \cdot F(n-2)$

Q: Prove this algorithm returns $3^n - 2^n$ for all $n \ge 0$.

Let $P(n)$ be the predicate $F(n)$ returns $3^n - 2^n$. We will prove $P(n)$ is true for all $n \ge 0$, using strong induction

$\boxed{\text{Base cases}}$: We will show $P(0)$ and $P(1)$. When the input is 0, we return 0. Since $3^0 - 2^0 = 1 - 1 = 0$, this is correct. When the input is 1, we return 1. Since $3^1 - 2^1 = 3 - 2 = 1$, this is correct.

Inductive step: Let $\boxed{k \ge 1}$. Assume $P(j)$ is true for all j such that $\boxed{0 \le j \le k}$. We will prove $P(k+1)$

We want $k+1$ to be larger than base cases, so choose k to be larger than or equal to largest base case

Want to assume all base cases are true, so j starts at smallest base case.

We need to prove $P(0)$ and $P(1)$. Otherwise when try to prove $P(2)$, look at $f(2-1) = f(1)$ and $f(2-2) = f(0)$, need to assume these output correctly