

Calculating Probability

What is the probability of (2 dice sum to 9)?

1. Define the sample space

set of all possible outcomes

think big!

$$S = \{(1,1), (1,2), (1,3) \dots (6,6)\}$$

$$= \{1,2,3,4,5,6\}^2$$

2. Define the event

subset of sample space you care about

$$E = \{(6,3), (3,6), (5,4), (4,5)\}$$

3. Ask: are all elements of sample space equally likely?

Yes

$$\Pr(E) = \frac{|E|}{|S|}$$

every outcome is equally likely

$$|E| = 4$$

$$|S| = 6 \times 6$$

(product rule)

No

$$\Pr(E) = \sum_{i \in E} \Pr(i)$$

Probability of outcome i

Sum over all elements in E

Note $\sum_{i \in S} \Pr(i) = 1$

$$\Pr(2 \text{ dice add to } 9) = \frac{4}{36} = \frac{1}{9}$$

Useful fact: $\Pr(S-E) = 1 - \Pr(E)$

↑
not E

(sometimes easier to calculate not E)

Q: Lottery where a 4-digit number is chosen randomly. What is the size of the sample space?

A) 40 B) 10,000 C) $\binom{10}{4}$

↑

Use product rule!

$$10 \cdot 10 \cdot 10 \cdot 10$$

Suppose you win some money if you get 3 of 4 numbers matching.

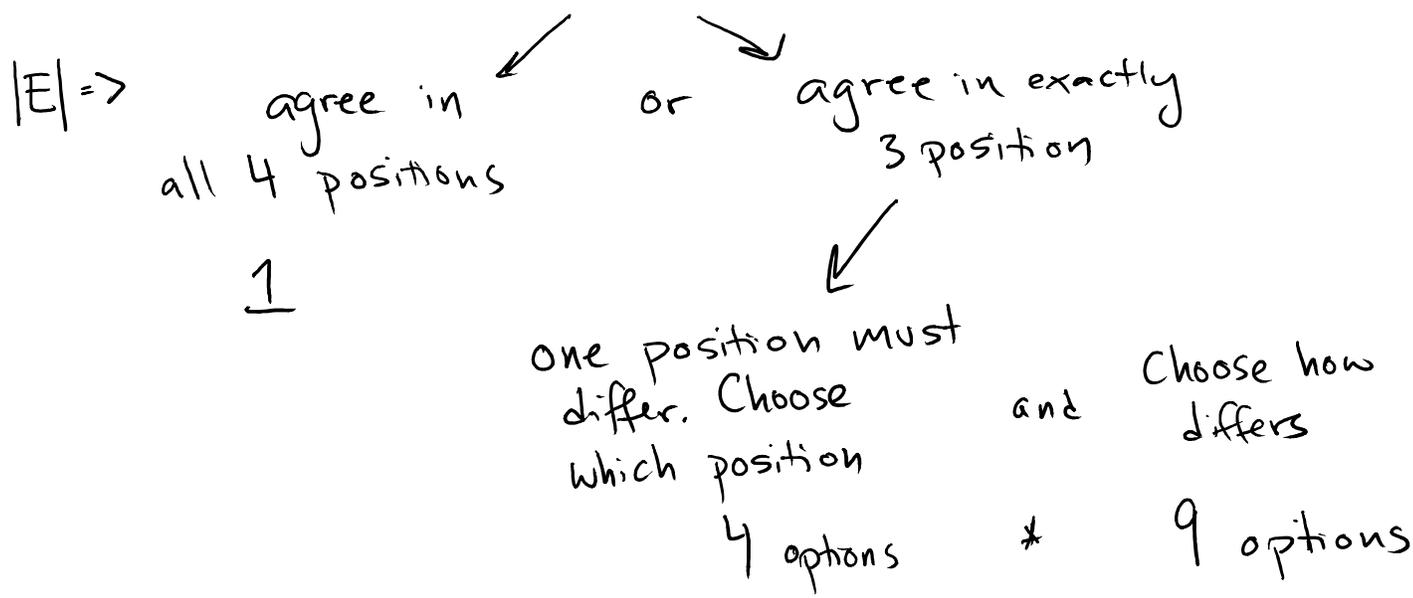
What is the probability you don't win if you buy 1 ticket with # 1313.

1. Sample Space: $\{1, 2, 3, \dots, 10\}^4$

2. Event (I'll calculate probability of winning and then do $1 - \Pr(E)$.)

$$E = \{x: x \in \{1, 2, \dots, 10\}^4 \wedge x \text{ agrees with } 1313 \text{ in at least } 3 \text{ positions}\}$$

3. Use formula $|E|/|S|$ b/c all outcomes are equally likely



$$1 + 4 \cdot 9 = 37$$



$$\Pr(\bar{E}) = 1 - \frac{37}{1000} = .9963$$

for example

- 1 3 1 3
- 2 3 1 3
- 3 3 1 3
- 4 3 1 3
- 5 3 1 3
- 6 3 1 3
- 7 3 1 3
- 8 3 1 3
- 9 3 1 3

9 options
Differ from 0 3 1 3 in first position