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Problem:

How to figure out $Pr(i)$ in more complex situations?

Then the probability of $E \subseteq S$ is

$$Pr(E) = \sum_{i \in E} Pr(i)$$

means add up $Pr(i)$ for all elements $i \in E$

Often you are given information about other events. Not the one you care about. For example:

ex: You are in a quidditch series against Skidmore. First team to win 2 games is champion

- Midd has $1/2$ chance of winning first game
- If Midd won the previous game, they have a $2/3$ chance of winning next.
- If Midd lost the previous game, they have a $1/3$ chance of winning next.

What is the probability Midd wins?

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Sample Space: $\{WW, WLW, \ldots$

win first

win second

win first

lose second

win third

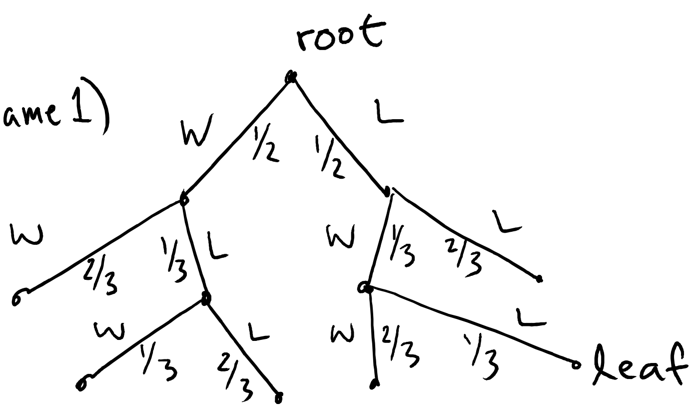
What is $Pr(WLW)$?

Use tree:

Option 1 (Game 1)

Option 2 (Game 2)

Option 3 (Game 3)



$\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{18}$

Sample Space: are all paths from root to leaves:

$S = \{WW, WLW, WLL, LWW, LWL, LL\}$

To calculate probability of outcome, multiply weights on path from root to leaf

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Math behind that strategy:

$$\Pr(E \cap F) = \Pr(E|F) \cdot \Pr(F)$$

Probability both event E and event F occur

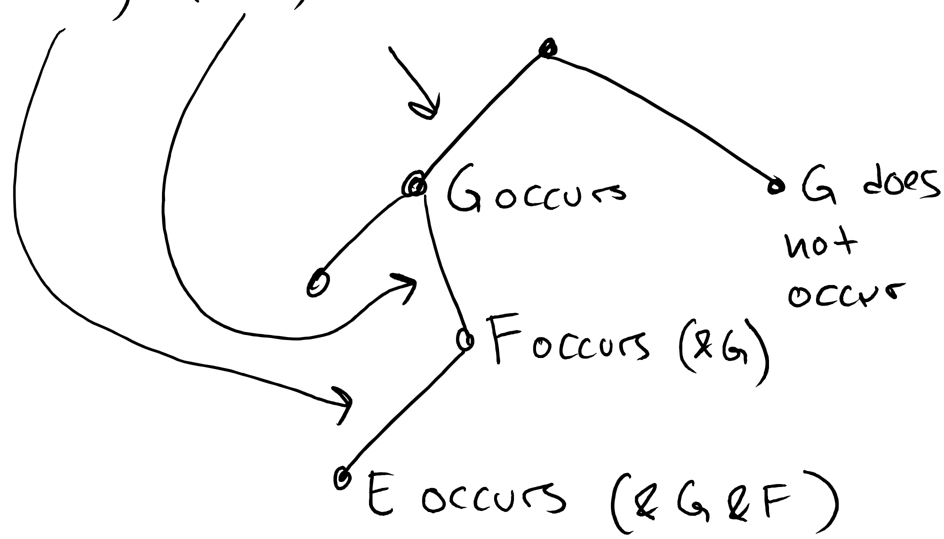
Probability of event E happening <u>if</u> you know event F happened

Probability event F occurs

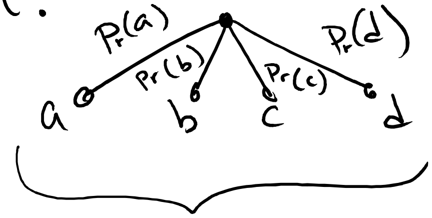
$$\Pr(\cdot|\cdot) = \text{"conditional probability"}$$

Can Chain together

$$\Pr(E \cap F \cap G) = \Pr(E|F \cap G)\,\Pr(F|G)\,\Pr(G)$$

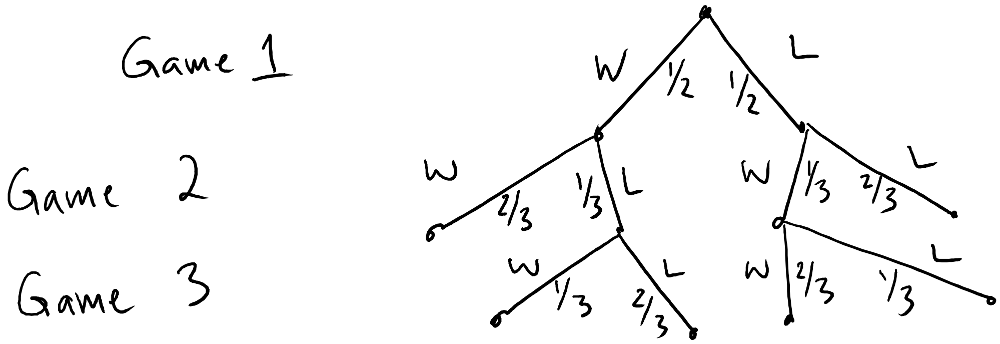


Important:



$$\Pr(a) + \Pr(b) + \Pr(c) + \Pr(d) = 1$$

represent different options. No overlap!



Event Midd Wins = $\{WW, WLW, LWW\}$

$Pr(\text{Midd Wins}) = Pr(WW) + Pr(WLW) + Pr(LWW)$

- $Pr(WW) = 1/2 \cdot 2/3 = 1/3$
- $Pr(WLW) = 1/2 \cdot 1/3 \cdot 1/3 = \frac{1}{18}$
- $Pr(WLL) = 1/2 \cdot 1/3 \cdot 2/3 = \frac{1}{9}$

$Pr(\text{Midd Wins}) = 1/3 + 1/18 + 1/9 = 1/2$

See slides for additional problems/solutions

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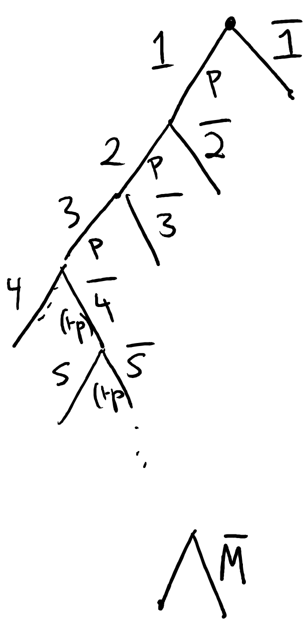
While tree approach always works, tree gets big, fast.

Most of the time, instead of writing tree out, imagine the tree in your mind, and use it to calculate probabilities.

Then use counting rules

ex: If we label edges of a graph $1, 2, 3, \ldots M$, and include each edge with probability p, what is the probability that only edges 1, 2, and 3 are present?

Imagine Tree:



$$\boxed{p^3(1-p)^{M-3}}$$

Any 3 edges?

How many paths: $\binom{M}{3}$ ← ways to choose which 3 edges to include. Each path has prob $p^3(1-p)^{M-3}$.

$$\Pr(\text{Any 3 edges}) = \binom{M}{3} p^3 (1-p)^{M-3}$$