

CS200 - Problem Set 3

Due: Monday, Oct. 2 to Canvas before class

Please read the sections of the syllabus on problem sets and honor code before starting this homework.

1. *Quantifiers*

Consider the following statement:

$$\forall x, \exists y : (y > x) \wedge (\forall z, ((z \neq y) \wedge (z > x)) \rightarrow (z > y))$$

- (a) **[3 points]** If the domain of x , y and z is \mathbb{Z} , state whether the statement is true or false. Justify your answer.
- (b) **[3 points]** If the domain of x , y and z is \mathbb{R} , state whether the statement is true or false. Justify your answer.

2. *Turning English Into Math* **[3 points each]**

When writing a proof, it is often helpful to use mathematical notation, rather than writing out the equivalent in English. This question will help you to practice this skill.

Let S be a set of students in a class and $f(s)$ be the score obtained by student s in an exam. Translate the English description of each predicate or statement below into a logical formula using quantifiers. When writing a formula for a statement or a predicate, you may use any propositions/predicates that you have previously defined. Use the \equiv symbol in your answer. For example, the for a , you should write $H(n) \equiv \dots$

To get full credit, your answer should only use mathematical notation, and your response should be approximately as concise as mine.

- (a) Predicate $H(n)$ asserts: n is the highest score that any student got on the exam.
- (b) Predicate $B(s)$ asserts: student s got the highest score.
- (c) Statement p asserts: at least two students got the highest score.
- (d) Predicate $R(s)$ asserts: student s got 10 points less than the highest score.
- (e) Statement t asserts: the second highest score in the class is 10 points less than the highest score.

3. *Implications* **[3 points each]**

There are many ways to represent a logical implication (\rightarrow) in English. To make proofs more interesting to read, we often take advantage of these different ways of phrasing the same underlying mathematical statement. In the following, I will ask you to rewrite sentences in the form $p \rightarrow q$. For example, “I get a brain freeze if I eat ice cream” should be rewritten “I eat ice cream \rightarrow I get a brain freeze.” Try to reason these out based on the English meaning first. If you are having trouble, check out p. 43 of Book of Proof.

- (a) I open my umbrella whenever it rains.
- (b) I miss class only if I am unwell.
- (c) You can't invent unless you are curious and knowledgeable.

4. *Logical Equivalences* [**3 points each**] The following are important logical equivalences. (They are worth memorizing!)

- (a) $p \rightarrow q \equiv \neg q \rightarrow \neg p$ (a statement is equivalent to its contrapositive)
- (b) $\neg(p \wedge q) \equiv \neg p \vee \neg q$ (DeMorgan's Law)
- (c) $\neg(p \vee q) \equiv \neg p \wedge \neg q$ (DeMorgan's Law)
- (d) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ (\wedge distributes over \vee)
- (e) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ (\vee distributes over \wedge)

Give an informal justification for each of (a), (b), and (c). Show (d) through truth table. When writing the truth table, be sure to include columns for subexpressions, such as $p \wedge q$. The following is an example truth table for (e) showing in the last column the equivalence of output columns 2 and 5.

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$	$p \vee (q \wedge r) \leftrightarrow (p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F	T
F	F	T	F	F	F	T	F	T
F	F	F	F	F	F	F	F	T

5. *Deduction* [**6 points**]

You meet two spiders on the road. Everyone knows that a spider either always tells the truth, or always lies. The first spider says, "If we are brothers, then we are both liars." The second spider says, "We are cousins or we are both liars." Are both spiders telling the truth?

6. *Induction Proofs* [**0 points - do this for practice as needed. Most of these problems have solutions.**]

- (a) BOP 10.1
- (b) BOP 10.11
- (c) DMOI 2.5.5
- (d) DMOI 2.5.11
- (e) Choose another algorithm from coding bat to prove is correct.

7. How long did you spend on this homework?