CS200 - Problem Set 2

Due: Monday, Sep. 25 to Canvas before class

Please read the sections of the syllabus on problem sets and honor code before starting this homework.

1. Set Builder to Roster Notation [2 point each]

The following sets are described in set builder notation. Describe each of them in roster notation, instead. The following symbols are used: \mathbb{Z} denotes the set of integers; \mathbb{R} denotes the set of real numbers; \mathbb{N} denotes the set of natural numbers, i.e., $\mathbb{N} = \{0, 1, 2, ...\}$.

- (a) $\{r \mid r \in \mathbb{R} \text{ and } r = r^2\}$
- (b) $\{n \mid n \in \mathbb{N} \text{ and } n > n^2\}$
- (c) $\{x \mid x \text{ is a letter in the word } accommodate\}$
- (d) $\{z^2 \mid z \in \mathbb{Z} \text{ and } 6 < z^3 < 160\}.$
- (e) $\{S \subseteq \{2, 4, 6, 8\} \mid S \cap \{2, 4\} \neq \emptyset \text{ and } |S| \text{ is even}\}$

2. Set builder notation [3 points each]

Write each of the following sets using set-builder notation:

- (a) $\{\ldots, 1/8, 1/6, 1/4, 1/2, 2, 4, 6, 8 \ldots\}$
- (b) Express the set of all sets of 2 integers such that the two numbers in the set are non-zero, have opposite signs, and the magnitude of one of them is the square of the magnitude of the other.

3. Universal Set [2 points]

Let U, the universal set, be the set of even integers from 2 to 12 inclusive, and let $A = \{4, 6, 7, 9\}, B = \{2, 3, 4, 5, 7\}$. What is $\overline{A - B}$?

- 4. Set Operations [2 points each] Simplify each of the following expressions, where A is an arbitrary set, \emptyset is the empty set, and U is the universal set. Hint: each answer to (a)-(h) is one of A, U, or \emptyset . Just write down the answer: no proof needed, no steps need be shown.
 - (a) $A \cap U$
 - (b) $A \cap \emptyset$
 - (c) $A \cup U$
 - (d) $A \cup \emptyset$
 - (e) $A \cup A$

- (f) $A \cap A$
- (g) $A \cup \overline{A}$
- (h) $A \cap \overline{A}$
- 5. *Statements* **[3 points each]** For each of the following sentences, decide whether it is a statement, predicate, or neither, and explain why
 - (a) Call me Ishmael.
 - (b) The universe is supported on the back of a giant tortoise.
 - (c) x is a multiple of 7.
 - (d) The next sentence is true.
 - (e) The preceding sentence is false.
 - (f) The set \mathbb{Z} contains an infinite number of elements.
- 6. Statements [2 point each] Simplify each of the following expressions, where p denotes a statement, and T and F are the Boolean constants *true* and *false*. Hint: each answer is one of p, T, or F. No proof needed, no steps need be shown.
 - (a) $T \wedge p$
 - (b) $F \wedge p$
 - (c) $T \lor p$
 - (d) $F \lor p$
 - (e) $p \lor p$
 - (f) $p \wedge p$
 - (g) $p \lor \neg p$
- 7. How long did you spend on this homework?