

CS200 - Problem Set 10

Due: Monday, Dec. 4 to Canvas

1. In this problem you will show that m is asymptotically bounded below by n , and asymptotically bounded above by n^2 , for connected graphs.
 - (a) **[11 points]** Prove that if you have an undirected connected graph (i.e. there is a path from every vertex to every other vertex) with n vertices and m edges, then $m = \Omega(n)$. (Assume there is at most one edge between every pair of vertices and no self-loops. Hint: prove $m \geq n - 1$.)
 - (b) **[3 points]** Explain why if you have an undirected connected graph with n vertices and m edges, then $m = O(n^2)$. (Assume there is only one edge between every pair of vertices and no self-loops.)
2. **[11 points]** Graph coloring is an important problem in computer science. The goal is to assign to each vertex in a graph a color, but you can't assign the same color to two vertices if they are adjacent. For example, if you create a vertex for each country in the world, and put an edge between two vertices if they share a border, then a solution to graph coloring would tell you how to color the countries so that no two adjacent countries are the same color. The minimum possible number of colors needed to color a graph is called the *chromatic number* of the graph. Prove that the chromatic number of a tree with at least two vertices is 2.
3. **[6 points]** Create finite state machine (either draw a graph, or create a table) that models a toll machine that takes quarters, nickels, and dimes. The toll opens if at least 25 cents have been entered, and then should reset for the next person. If a person overpays, the machine eats their money. Sad :(
4. **[6 points]** Draw a DFA that accepts all strings over $\{0, 1\}^*$ that contain an even number of 0s.
5. **[6 points]** Draw a DFA that accepts all strings over $\{0, 1\}^*$ that contain an even number of 0s and an odd number of 1s.
6. **[6 points]** Draw a DFA that accepts all strings over $\{0, 1\}^*$ that start with two 0s and end with two 1s.
7. How long did you spend on this homework?