Last time
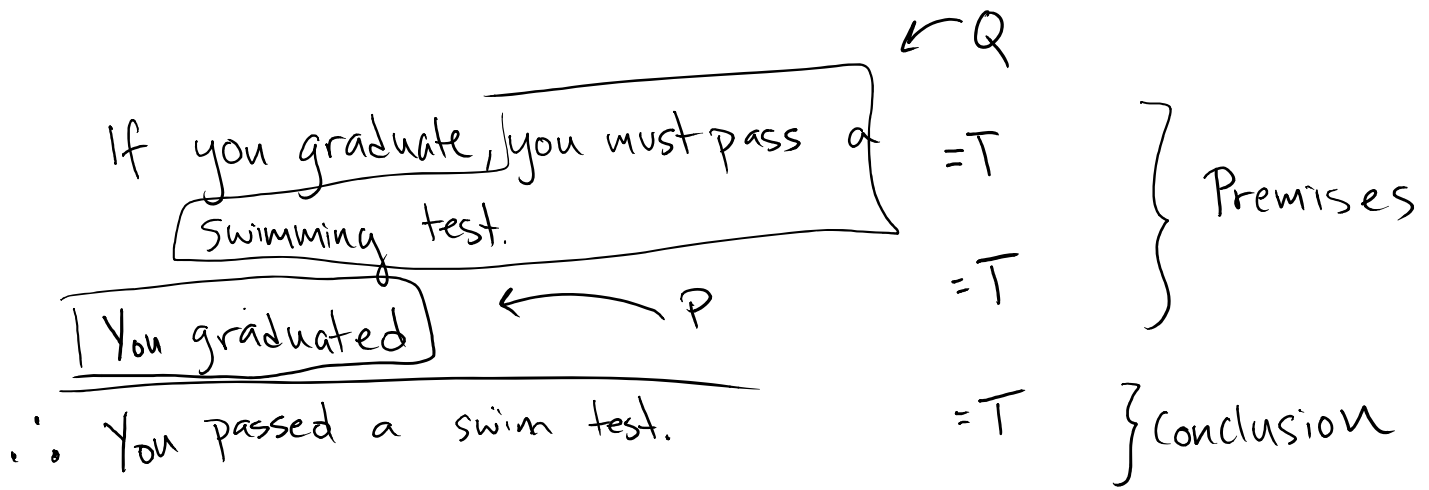
- Quantifiers
- Logical Equivalences $\quad (P \to Q) \equiv \neg P \vee Q$
- Tautology $\quad (P \to Q) \vee (Q \to R)$ statement is always true

Questions

Deductions : using known true statements to create new true statements

If you graduate, you must pass a swimming test. $\quad \leftarrow Q$ $= T$ $\Big\}$ Premises

You graduated $\quad \longleftarrow P$ $= T$

∴ You passed a swim test. $\quad = T \quad \}$ conclusion

$$P \to Q$$
$$\frac{P}{\therefore Q}$$

If $P \to Q$ is true and P is true, then Q must be true

If premises are true, conclusion is true. If premises are false, conclusions don't hold.

This is like inductive Proof! $P(K)$ is true $\to P(K+1)$ is true

graduate pass swim test

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

Deduction Problem

Holmes owns two suits: one black and one tweed. He always wears either a tweed suit or sandals. Whenever he wears his tweed suit and a purple shirt, he chooses to not wear a tie. He never wears the tweed suit unless he is also wearing either a purple shirt or sandals. Whenever he wears sandals, he also wears a purple shirt. Yesterday, Holmes wore a bow tie. What else did he wear?

(Assume he wears some suit.)

1. Label Statements with letters. Write down true statements
2. Write truth table, and DEDUCE!

- W = Holmes wears a tweed suit
- P = Holmes wears a purple shirt
- S = Holmes wears sandals
- T = Holmes wears a tie

True statements:

W ∨ S
(W ∧ P) → ¬T
W→(P ∨ S)
S → P

Since ¬T is false, for this statement to be true, we need W ∧ P to be false

W	S	P	W ∨ S	(W ∧ P) → ¬T	W→(P ∨ S)	S → P
T	T	T	T	F	T	T
T	T	F	T	T	T	F
T	F	T	T	F	T	T
T	F	F	T	T	F	T
F	**T**	**T**	**T**	**T**	**T**	**T**
F	T	F	T	T	T	F
F	F	T	F	T	T	T
F	F	F	F	T	T	T

- Then if we cross out all rows where one of the final four columns contains an F, we are only left with one row, the one in bold!

Deduction Problem

9. Tommy Flanagan was telling you what he ate yesterday afternoon. He tells you, "I had either popcorn or raisins. Also, if I had cucumber sandwiches, then I had soda. But I didn't drink soda or tea." Of course you know that Tommy is the worlds worst liar, and everything he says is false. What did Tommy eat?

Justify your answer by writing all of Tommy's statements using sentence variables (P, Q, R, S, T), taking their negations, and using these to deduce what Tommy actually ate.

1. Label Statements with letters. Write down true statements
2. DEDUCE! (Without a truth table.)

- P = Tommy at popcorn
- R = Tommy ate raisins
- C = Tommy ate cucumber sandwiches
- T = Tommy drank tea
- S = Tommy drank soda

True statements:

$\neg(P \lor R)$
$\neg(C \to S)$
$\neg(\neg(S \lor T))$

The only way $\neg(C \to S)$ is true is if $(C \to S)$ is false. $(C \to S)$ is false if C is true and S is false. The only way $\neg(P \lor R)$ is true is if $(P \lor R)$ is false. $(P \lor R)$ is false if P is false and R is false. $\neg(\neg(S \lor T))$ is true if $S \lor T$ is true. But since S is false, we must have T is true. Therefore, Tommy ate cucumber sandwiches with tea.